MHD Simulations for ITER

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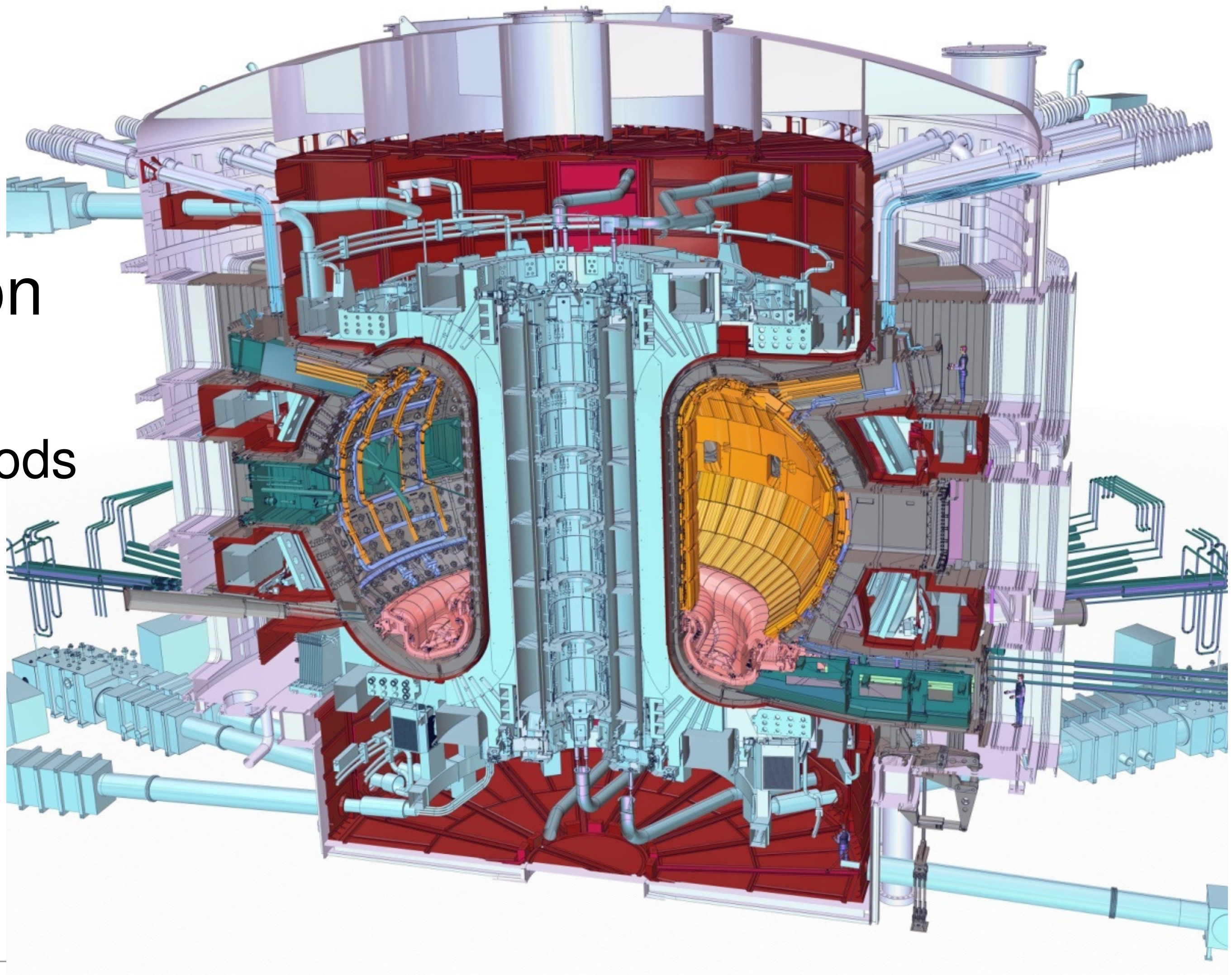
ITER Organization

Disclaimer: The views and opinions expressed herein do not necessarily reflect those of the ITER Organization

Outline

- MHD in ITER
  - Disruptions
  - ELMs
- MHD Simulation
  - JOREK
    - Numerical methods
- Applications
  - ELMs
  - VDE
  - QH-mode
- Conclusion



Magneto-Hydro-Dynamics (MHD)

- Theoretical model (H. Alfven, 1942)
  - description of the plasma as a electrically conducting fluid embedded in a magnetic field
    - Combining Navier-Stokes fluid equations with (pre-) Maxwell's equations
      - Conservation of mass, momentum, energy and flux
      - 19^{th} century physics model
  - applications: plasmas, solar physics, astrophysics, magnetosphere, dynamos, …

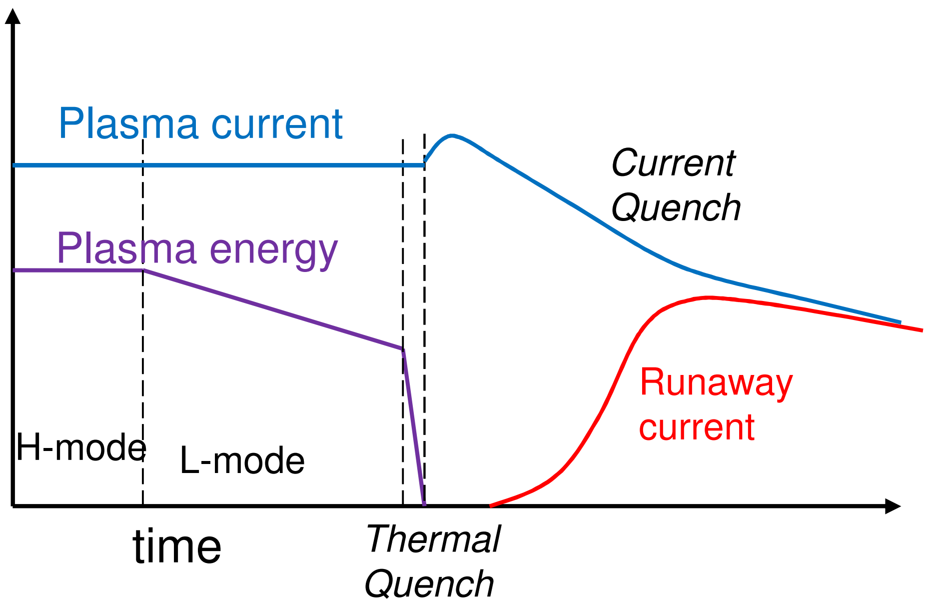
- MHD in tokamak plasmas
  - refers to large scale, magnetic, instabilities in the plasma
    - Driven unstable by pressure gradients and current (gradients)
      - Limits global pressure and current and local pressure, current gradients
  - accurately described by the MHD model

MHD Instabilities in ITER

- Disruptions, Vertical Displacement Events (VDEs)
  - Sudden termination of plasma, leading to high heat loads and electro-mechanical forces
- Edge Localised Modes (ELMs)
  - Repetitive MHD instabilities at the plasma edge leading to enhanced erosion of the plasma facing components (divertor)
- Neoclassical Tearing Modes (NTMs)
  - Magnetic islands driven by plasma pressure, leading to a reduction in the energy confinement (reduced plasma performance)
- Sawteeth
- Resistive wall modes
- Fast particle driven modes (Alfven eigenmodes)

ITER Disruptions

- Sequence of events:



- Major Disruptions (MD)
  - locked modes
  - density limit
  - beta limit
- Vertical Displacement Event (VDE, up/down)
  - loss of vertical position control

- The largest thermal loads occur Thermal Quench (TQ, 1-3ms)
- Major mechanical forces act on plasma facing components during Current Quench (CQ, 50-150ms)
- Runaway electrons will be generated during Current Quench

MHD Control in ITER

- Control of MHD instabilities in ITER is essential:
  - For machine protection:.
    - Transient heat and mechanical loads due to **disruptions** and **ELMs** are acceptable in current tokamaks but need to be controlled / avoided / mitigated in ITER
  - For performance optimization:
    - In the baseline scenario, to obtain Q=10 (at β_N~1.8 with large sawteeth) **Neoclassical Tearing modes** (NTM) need to be suppressed. **Sawteeth** may also need to be controlled.
    - In advanced steady state scenario at Q=5 (at β_N~3.0), **Resistive Wall Modes** (RWM) will need to be stabilized to operate above the no-wall limit.
    - Fast particle driven **Alfven modes** may also need to be controlled

MHD Control Methods in ITER

- Disruptions
  - Massive gas injection, MGI (or massive material injection, MMI)
  - Valves, shattered pellet injector
- ELMs
  - Resonant Magnetic Perturbations (27 in-vessel coils)
  - Pellet injection (~40Hz)
  - Vertical kicks, <10MA
- Neoclassical tearing modes
  - Electron cyclotron Heating/current drive (ECRH/ECCD)
- Sawteeth
  - Ion cyclotron heating (ICRH), ECRH/ECCD
- Resistive wall modes
  - In-vessel coils

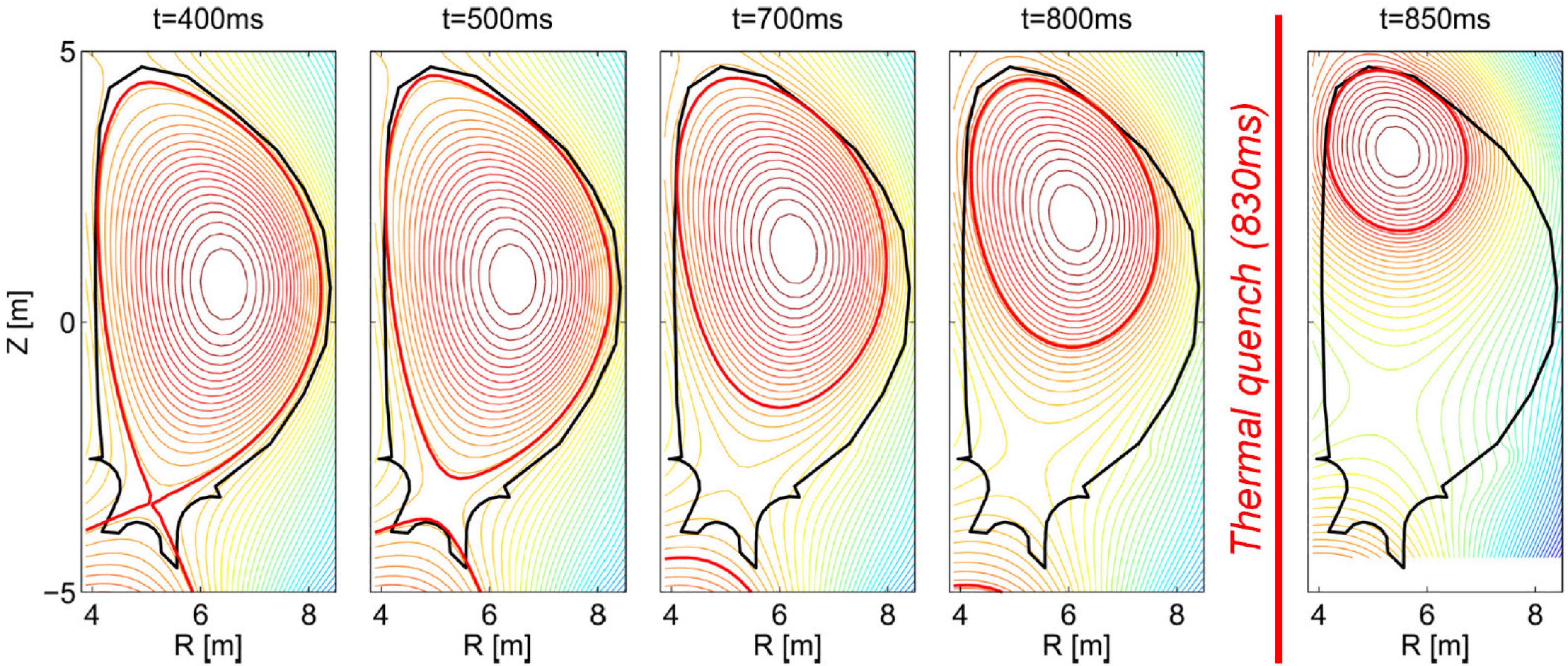
ITER Disruptions

- Heat loads:
  - Plasma thermal energy 350MJ, spread over 10-30m^2 , in 1-3 ms
    - Heat load ~ energy / (area x time$^{0.5}$) ~ 100-2000 MJ/m^{2}s$^{0.5}$
      - Melting of Tungsten at 50 MJ/m^{2}s$^{0.5}$ (2700^o C)

- Mitigation requires >90% radiation to spread power over walls
  - Injection of impurities (Ne, He, D_2, <1.8x10^{24} particles, gas and/or shattered pellets)
    - How to obtain an homogeneous distribution of radiation?
      - Number and position of injection points, amounts, time delay
    - Peaking factor of radiation influenced by MHD activity
    - Production of runaway electrons

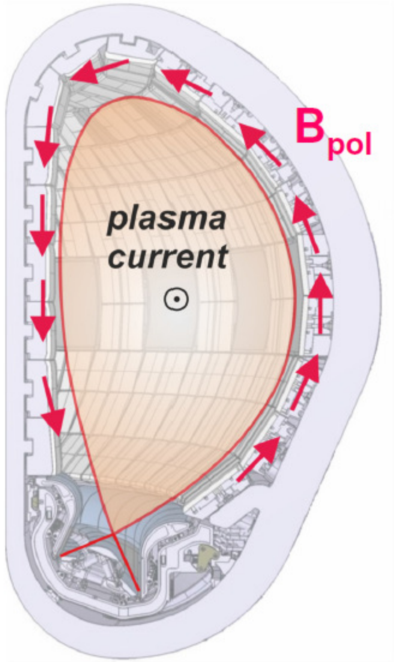
Vertical Displacement Events (VDE)

- Elongated plasmas are vertically unstable
  - feedback control keeps the plasma in place
  - In ITER, vertical displacements up to 16 cm can be controlled
    - **Failure of control leads to a vertical displacement event (VDE)**
      - time scale determined by magnetic field diffusion through resistive wall

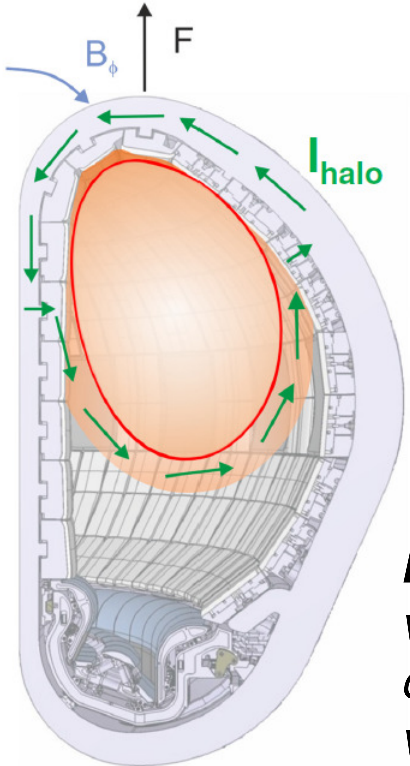


Wall Currents

- Movement of the plasma and decay of the plasma current cause a time variation of the magnetic field in the walls
  - Leading to induced currents in the metallic structures : **eddy currents**
- Direct contact of the plasma with the wall leads to a "current sharing"
  - Plasma current (partially) flows into the wall and back: **halo currents**
- Axi-symmetric VDEs lead to large vertical forces on the vessel
  - ~1000 tons in ITER



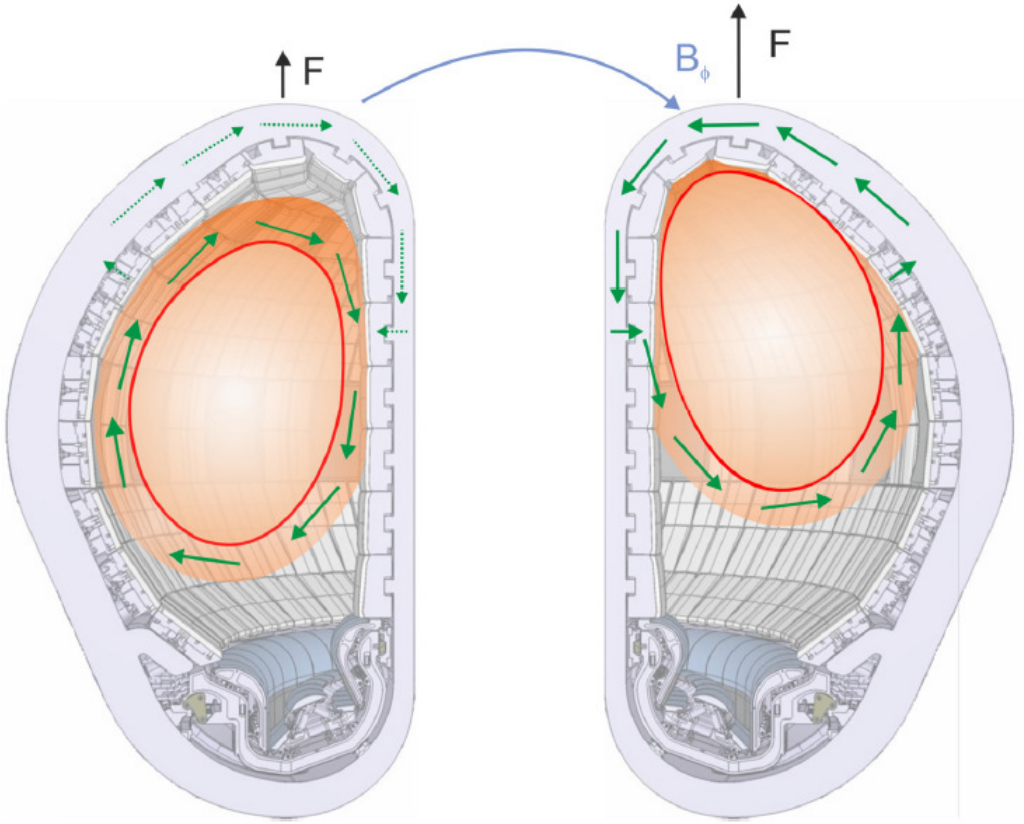
eddy currents *forces on the in-vessel structures*



halo currents *vertical forces on the vacuum vessel*

Asymmetric VDEs

- The shrinking of the plasma during a VDE can destabilize additional instabilities (kink modes)
  - Leading to asymmetric VDEs, vertical and sideways forces
  - Mode rotation may lead to resonant amplification, increasing the forces
    - **Physics basis for expected behavior in ITER is high priority**



Runaway Electrons

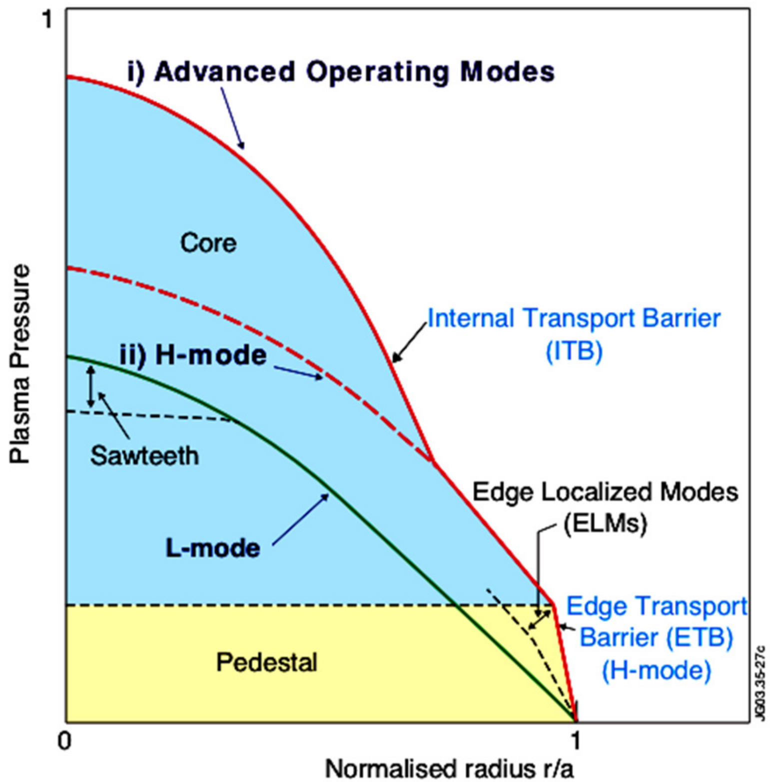
- During the Thermal Quench (1-3ms) the electron temperature drops from ~10 keV to ~10 eV
  - Increase in plasma resistivity (~$T^{3/2}$) leads to a large electric field
    - Electric field ~ resistivity x current density ~ 20V/m

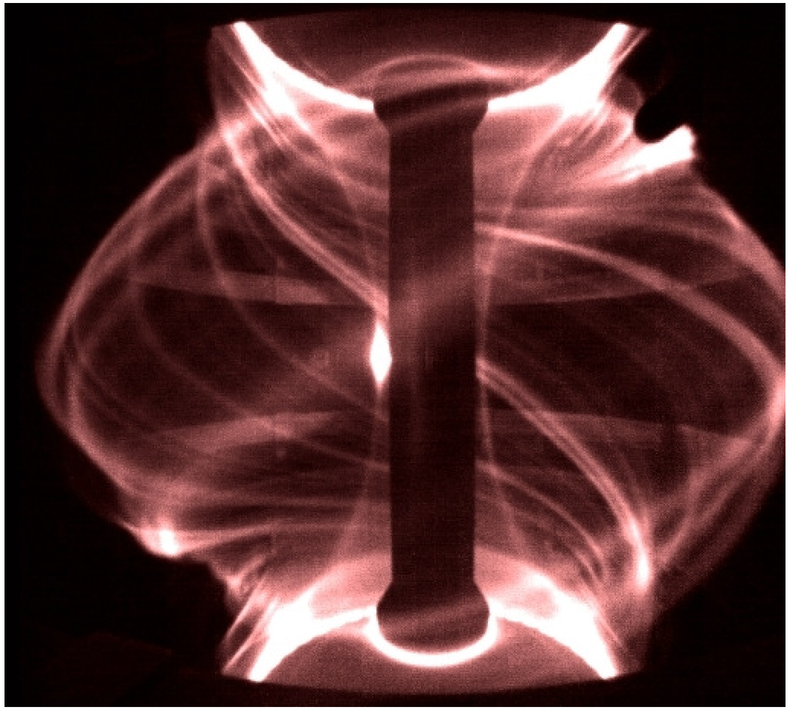
- Friction forces decrease with increasing electron energy
  - **Electric field accelerates electrons to relativistic speeds**
    - Runaway electrons : high energy electron beam

- ITER parameters: I_{RE} < 10MA, electron energy ~ 20MeV
  - Beam energy ~ 10MJ (kinetic), 200MJ (magnetic)
    - Can lead to deep melting of Be first wall
    - What fraction of magnetic energy is lost?
  - Runaway electrons must be mitigated (I_{RE} < 2MA)
    - Massive gas injection

H-mode, Edge Localised Modes

- H-mode regime: spontaneous stabilisation of turbulence at the plasma edge leads to large pressure gradients in the outer ~5% of the plasma
  - standard operating regime in ITER
- Edge pressure gradient is limited by an MHD instability (ballooning mode)
  - Edge Localised Mode (ELM) removes up to 10% of the plasma energy in ~200 microseconds

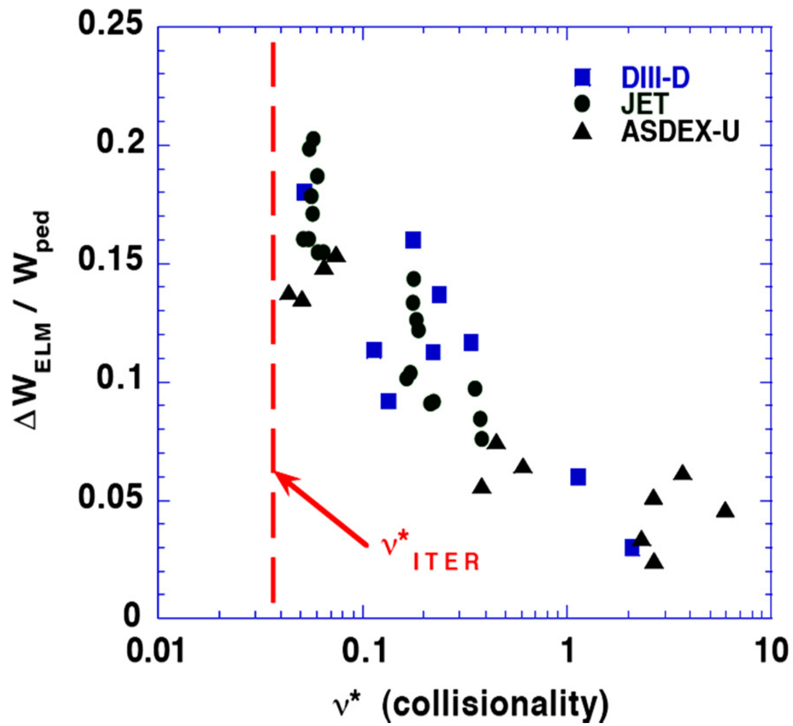




ELM in MAST [Kirk]

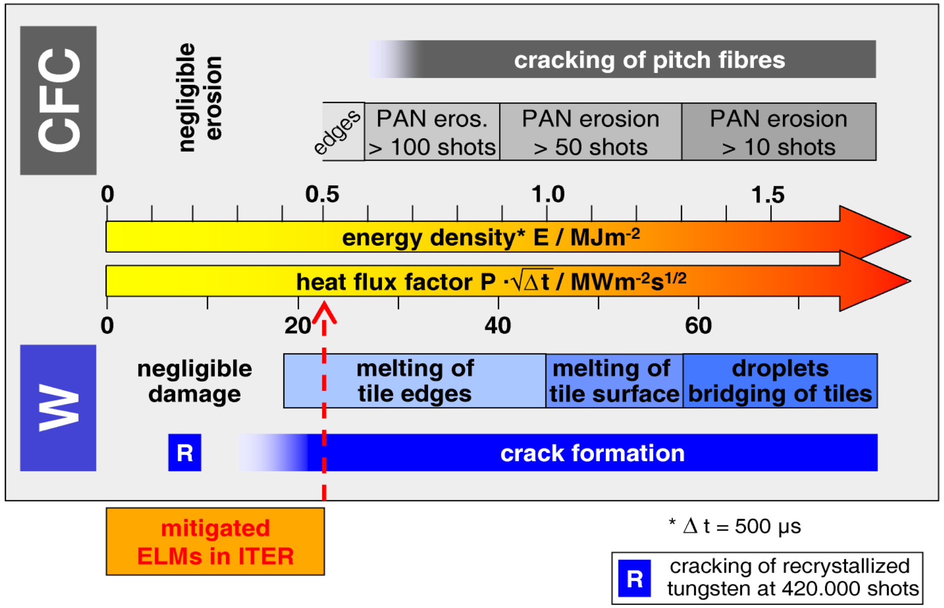
ELMs in ITER

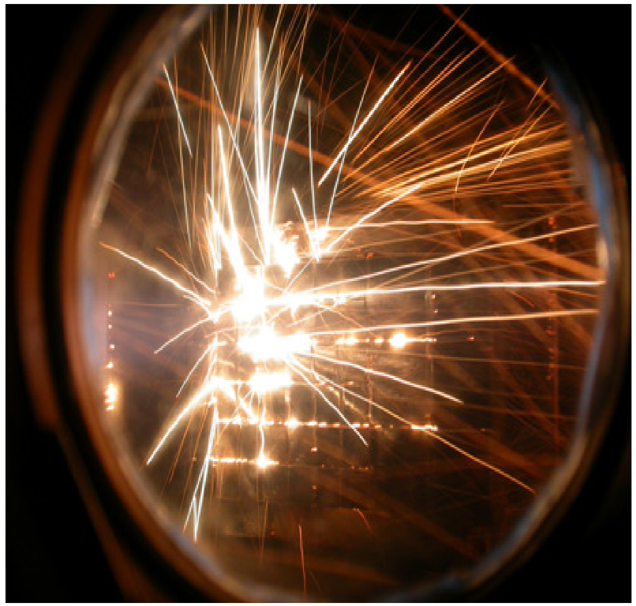
- Extrapolation form current experiments indicates natural ELMs in ITER will be very large: $\Delta W_{ELM}/W_{ped}$ ~ 0.2
  – W_{ped}~100-130MJ : ΔW_{ELM} ~ 20 MJ
    - Energy flux: ~10 MJ/m^2 (τ_{ELM}~250-500μs)
  – P_{sol}=140MW (x20-40%) : f_{elm}=1-3 Hz



Tolerable ELMs

- Experiments in plasma gun facilities (QSPA in Troitsk) and e-beams (Judith, FZJ):
  - max. energy flux of **0.5 MJm^{-2}** , no major reduction of the divertor lifetime
- Assume: no broadening + 2:1 in/out asymmetry + toroidal symmetry:
  - **ΔW_{ELM} ~ 0.7 MJ** , f_{ELM}~30-60Hz, 8-16x10^3 ELMs /Q=10 shot
  - mitigation **factor 30** required
  - unmitigated ELMs possible at lower plasma currents
    - 6-9MA, depending on footprint broadening



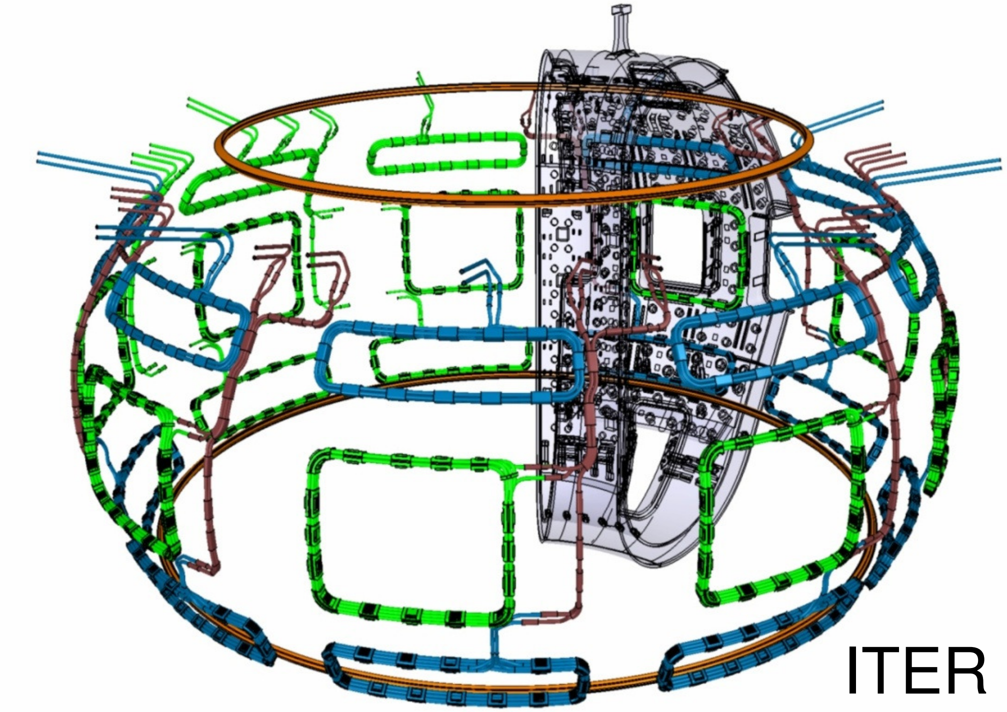


droplet ejection tungsten >1.4MJ

ITER ELM control

- Two main ELM control methods foreseen on ITER
  - Application of (Resonant) Magnetic Perturbations
    - ELM Stabilisation (or mitigation)
  - Injection of small pellets:
    - Triggering of ELMs at given “pellet pacing” injection frequency
      - Velocity 300-500m/s, Volume 17-34mm3, frequency 4-16Hz

- Other options:
  - Vertical Kicks
  - QH-mode(?)



ITER ELM coils

MHD Equations

- Resistive MHD equations:
  - Evolution of density, velocity, temperature and magnetic field

$$\partial_t \rho = -\nabla \cdot (\rho \mathbf{v}) + \nabla \cdot (\mathsf{D} \nabla \rho) + S_\rho,$$

$$\rho \partial_t \mathbf{v} = -\rho \mathbf{v} \cdot \nabla \mathbf{v} - \nabla(\rho T) + \mathbf{J} \times \mathbf{B} + \nu \nabla^2 \mathbf{v},$$

$$\partial_t T = -\mathbf{v} \cdot \nabla T - (\gamma - 1) T \nabla \cdot \mathbf{v} + \nabla \cdot (\bar{\mathsf{K}} \nabla T) + S_T,$$

$$\partial_t \mathbf{B} = -\nabla \times \mathbf{E} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \mathbf{J}),$$

$$\nabla \cdot \mathbf{B} = 0$$

  - Using vector potential:

$$\partial_t \mathbf{A} = \mathbf{v} \times (\nabla \times \mathbf{A}) - \eta (\nabla \times \nabla \times \mathbf{A})$$

Reduced MHD

- Formulation using electric potential (*u*) and magnetic flux (*ψ*):
  - Ansatz:

$$\vec{v} = -R\vec{\nabla}u(t)\times\vec{e}_\varphi + v_{\parallel}(t)\vec{B} \qquad \vec{B} = \frac{F_0}{R}\vec{e}_\varphi + \frac{1}{R}\vec{\nabla}\psi(t)\times\vec{e}_\varphi$$

Poloidal flux

$$\frac{1}{R^2}\frac{\partial\psi}{\partial t} = +\eta\nabla\cdot\left(\frac{1}{R^2}\nabla_\perp\psi\right) - \frac{1}{R}[u,\psi] - \frac{F_0}{R^2}\partial_\varphi u$$

parallel momentum

$$\vec{B}\cdot\left(\rho\frac{\partial\vec{v}}{\partial t} = -\rho\left(\vec{v}\cdot\vec{\nabla}\right)\vec{v} - \vec{\nabla}(\rho T) + \vec{J}\times\vec{B} + \mu\Delta\vec{v}\right)$$

poloidal momentum

$$\vec{e}_\varphi\cdot\nabla\times\left(\rho\frac{\partial\vec{v}}{\partial t} = -\rho\left(\vec{v}\cdot\vec{\nabla}\right)\vec{v} - \vec{\nabla}(\rho T) + \vec{J}\times\vec{B} + \mu\Delta\vec{v}\right)$$

Temperature

$$\rho\frac{\partial T}{\partial t} = -\rho v\cdot\nabla T - (\gamma-1)\rho T\nabla\cdot v + \nabla\cdot\left(\mathrm{K}_\perp\nabla_\perp T + \mathrm{K}_\parallel\nabla_\parallel T\right) + S_T$$

Density

$$\frac{\partial\rho}{\partial t} = -\nabla\cdot(\rho\vec{v}) + \nabla\cdot(D_\perp\nabla_\perp\rho) + S$$

- Extended to include diamagnetic and neoclassical flows

Boundary Conditions

- Plasma-wall interaction:
  - Wall is a strong pump for plasma
  - Fluid boundary conditions at the sheath entrance:
    - Parallel velocity:

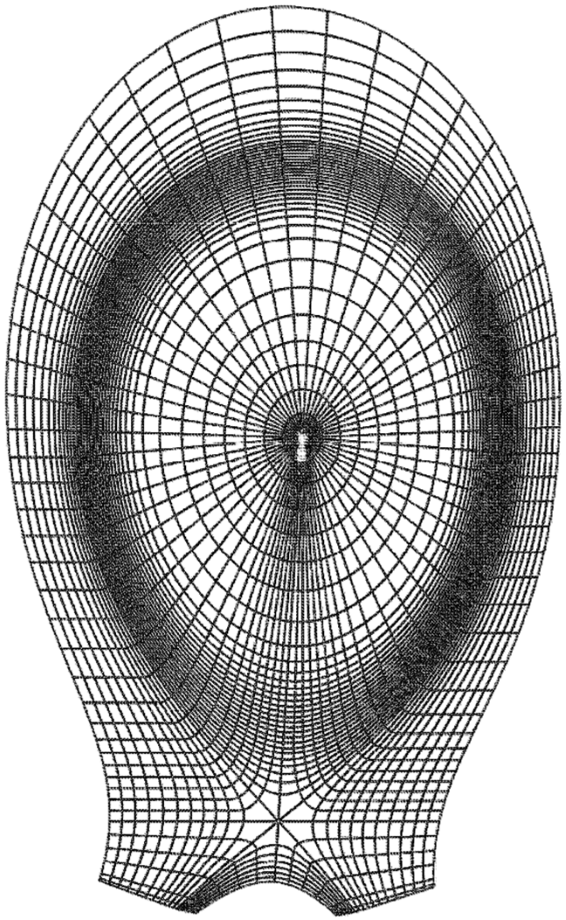
$$\frac{\vec{v}\cdot\vec{B}}{|B|} \geq c_s \qquad \vec{v}\cdot\vec{n} \geq \frac{\vec{B}\cdot\vec{n}}{|B|} c_s$$

    - Parallel energy flux: $nTv_{\|} + K_{\|}\nabla_{\|}T = \gamma_{sh}K_{\|}\nabla_{\|}T$
    - Potential: $e\phi = -T_e \ln\left(\frac{m_i}{2\pi m_e}\right)^{\frac{1}{2}}$

- Magnetic field:
  - Fixed boundary (ideal wall) : $\delta\vec{B}\cdot\vec{n}\big|_{wall} = 0$
  - Free boundary (resistive wall, vacuum, coils)
    - Continuity of total magnetic (electric) field

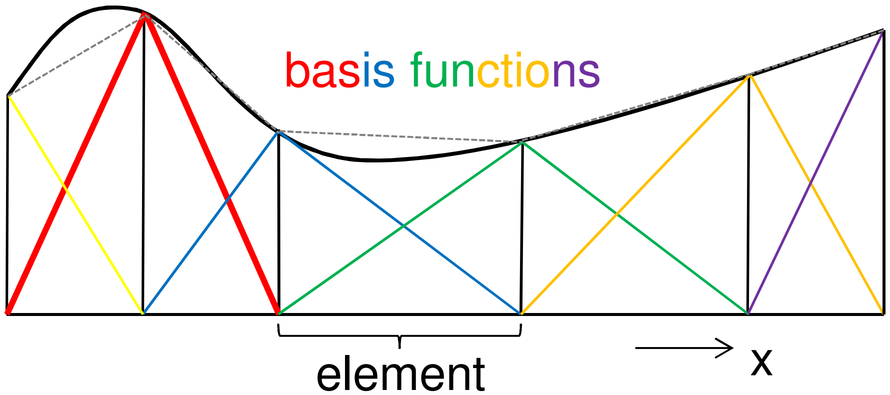
Non-linear MHD code JOREK

- Initial motivation: non-linear MHD simulations of Edge Localised Modes
  - Reduced MHD in toroidal geometry
  - Whole domain inside vacuum vessel, including open and closed field lines, x-point(s)
  - Divertor boundary conditions
  - Long time scales
- Evolving towards general MHD simulation code
  - Reduced and full (extended) MHD models
  - Including interaction with resistive walls, coils
  - JOREK team
- Characteristics:
  - C^1 iso-parametric Bezier finite elements (refinement)
    - real Fourier series in toroidal direction
  - Fully implicit time evolution
    - PaStiX sparse matrix solver
  - Parallelisation MPI-OPENMP
    - 256 - 2048 cpus

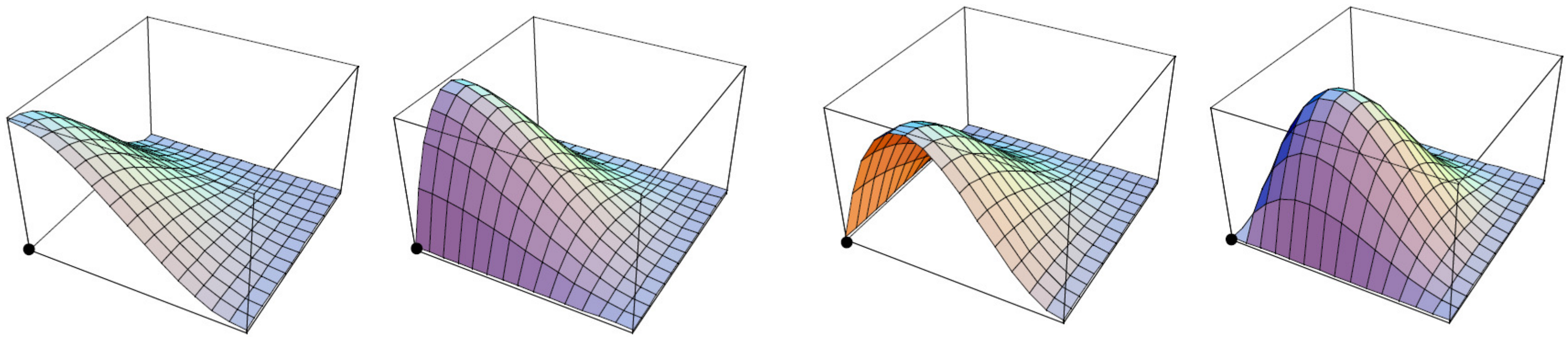


Finite Elements: Basis Functions

- Representation of variables using functions with local support (i.e. finite only in a small number of "elements")
  - Linear: $f(x) = \sum_{i=1}^{N} f_i H(x - x_i)$



  - Cubic Hermite: $f(x) = \sum_{i=1}^{N} f_i H(x - x_i) + f_i' h(x - x_i)$
    - C^1 continuity
    - In 2D: 4 unknowns (and basis functions) per node



Finite Elements: Weak Form

- Construct a weak form of the equation(s):

$$R^2\nabla\cdot\frac{1}{R^2}\nabla\psi=-FF'(\psi)-R^2p'(\psi) \qquad \psi(R,Z)=\sum_{i,j=1}^{N,M}\psi_{ij}H(R-R_i,Z-Z_j)$$

  – Multiply equation with each test function
    - Use test functions the same basis functions (Galerkin method)

$$\psi^*(R,Z)=H(R-R_i,Z-Z_j)$$

  – Integrate over volume:

$$\int\frac{\psi^*}{R^2}R^2\nabla\cdot\frac{1}{R^2}\nabla\psi dV=-\int\frac{\psi^*}{R^2}\left(FF'+R^2p'\right)dV$$

$$-\int\frac{1}{R^2}\nabla\psi^*\cdot\nabla\psi dV+\int\psi^*\frac{1}{R^2}\nabla\psi\cdot\vec{n}dA=-\int\frac{\psi^*}{R^2}\left(FF'+R^2p'\right)dV$$

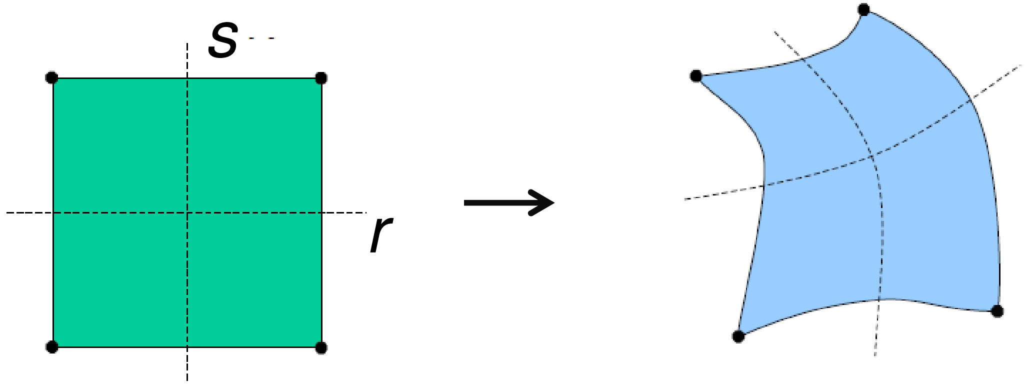
  – Gives a system of NxM equations for NxM unknowns

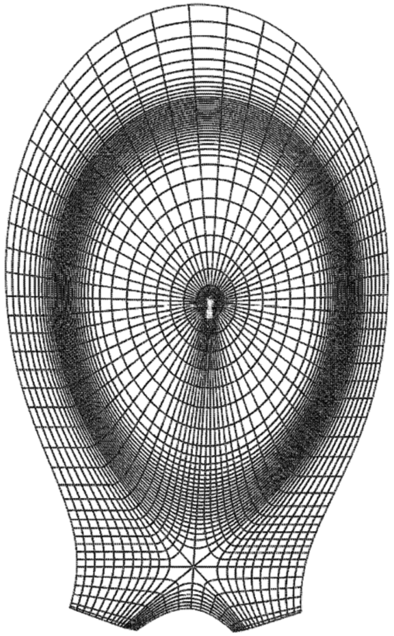
Iso-Parametric Finite Elements

- Anisotropy of MHD model between parallel and perpendicular directions
  - mode structures, heat conduction
  - advantageous to align finite elements with magnetic field (flux surfaces)
- Represent space with the same basis functions:
  - No loss of accuracy
  - Cubic Hermite:

$$x(r,s) = \sum_{4corners} x_i H_{00}(r,s) + \left.\frac{\partial x}{\partial r}\right|_i H_{10}(r,s) + \left.\frac{\partial x}{\partial s}\right|_i H_{01}(r,s) + \left.\frac{\partial^2 x}{\partial r \partial s}\right|_i H_{11}(r,s)$$

$$\psi(r,s) = \sum_{4corners} \psi_i H_{00}(r,s) + \left.\frac{\partial \psi}{\partial r}\right|_i H_{10}(r,s) + \left.\frac{\partial \psi}{\partial s}\right|_i H_{01}(r,s) + \left.\frac{\partial^2 \psi}{\partial r \partial s}\right|_i H_{11}(r,s)$$





Bezier Curves (1D)

- Bezier curves were defined by Pierre Bézier (1910–1999) at Renault in 1960s to describe parametrised curved surfaces
  - widely used in CAD-CAM, font definitions etc., OPENGL

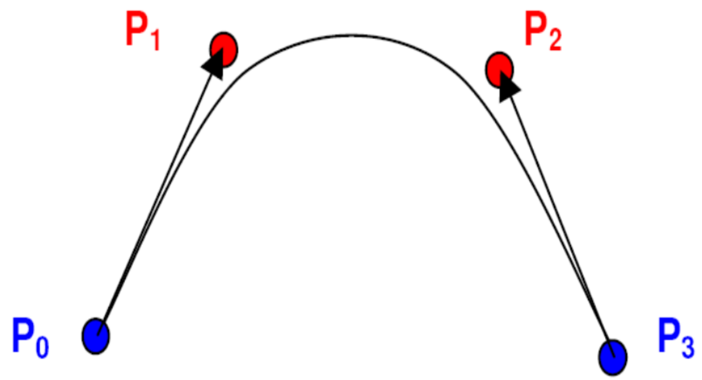
$$\vec{B}(t) = \sum_{k=0}^{N} \vec{p}_k \frac{N!}{k!(N-k)!} t^k (1-t)^{N-k} \qquad 0 < t < 1$$

- Cubic Bezier curve defined by 4 control points:

$$\vec{B}(t) = \vec{p}_0 (1-t)^3 + 3\vec{p}_1 t (1-t)^2 + 3\vec{p}_2 t^2 (1-t) + \vec{p}_3 t^3$$

- (naturally) Isoparametric:
  - Both space and variables are described by the same Bezier curves

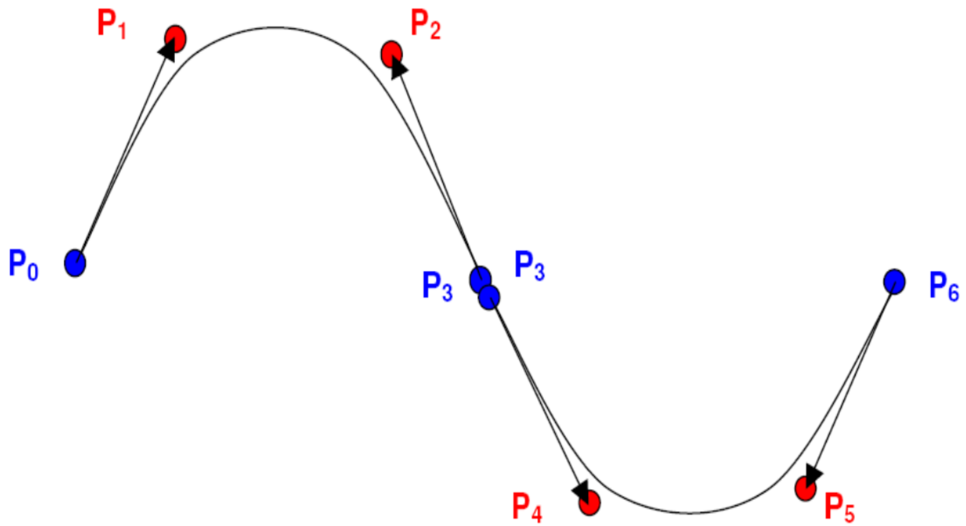
$$\vec{p}_k = (x, y, \psi)_k$$



C1 Continuity

- Continuity requirement for a 1D Bezier curve (in 2D or 3D space)
  - control points lie on a line:

$$\left(\vec{p}_2 - \vec{p}_3\right) = \alpha\left(\vec{p}_4 - \vec{p}_3\right)$$



- Physical variables and their first derivative are continuous in real space but not in the local coordinate
  - as opposed to cubic Hermite finite elements $\alpha = 1$

- Additional freedom allows local mesh refinement

Bezier Elements

- Redefine Bezier curves in terms of quantities defined at the nodes:
  - Scale factors (property of an element)

$$h_{32} = \left\| \vec{p}_2 - \vec{p}_3 \right\| \qquad h_{34} = \left\| \vec{p}_4 - \vec{p}_3 \right\|$$

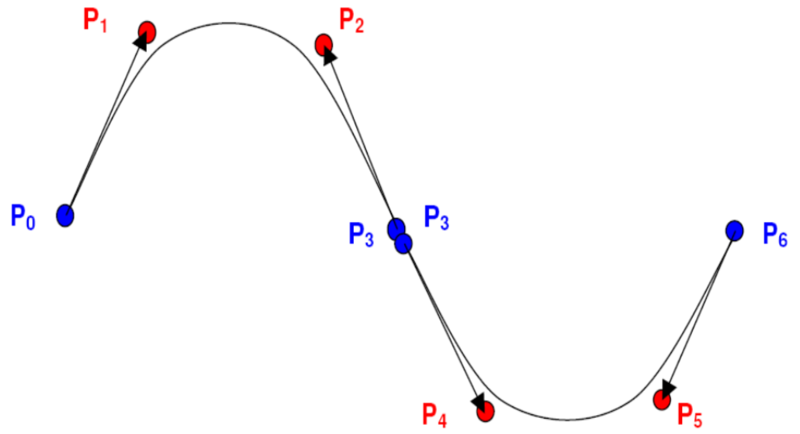
  - Unit vectors u_i (property of a node)

$$\vec{u}_3 = \frac{(\vec{p}_2 - \vec{p}_3)}{h_{32}} = -\frac{(\vec{p}_4 - \vec{p}_3)}{h_{43}}$$

  - Automatic C1 continuity
- Physical variables (unknowns):

$$\vec{p} = (R, Z, \psi)$$

$$h_{23} = \sqrt{(R_3 - R_2)^2 + (Z_3 - Z_2)^2} \qquad \vec{u}_i = \begin{pmatrix} \delta R_i \\ \delta Z_i \\ \delta \psi_i \end{pmatrix}$$



- Cubic Hermite finite elements are obtained with $h_{23}=h_{34}=1$
  - functions continuous in local coordinate (no refinement)

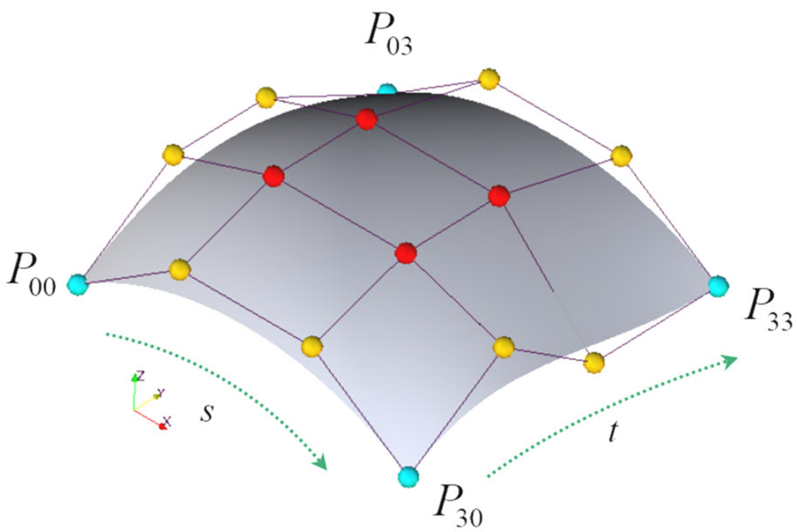
2D Bezier Patches

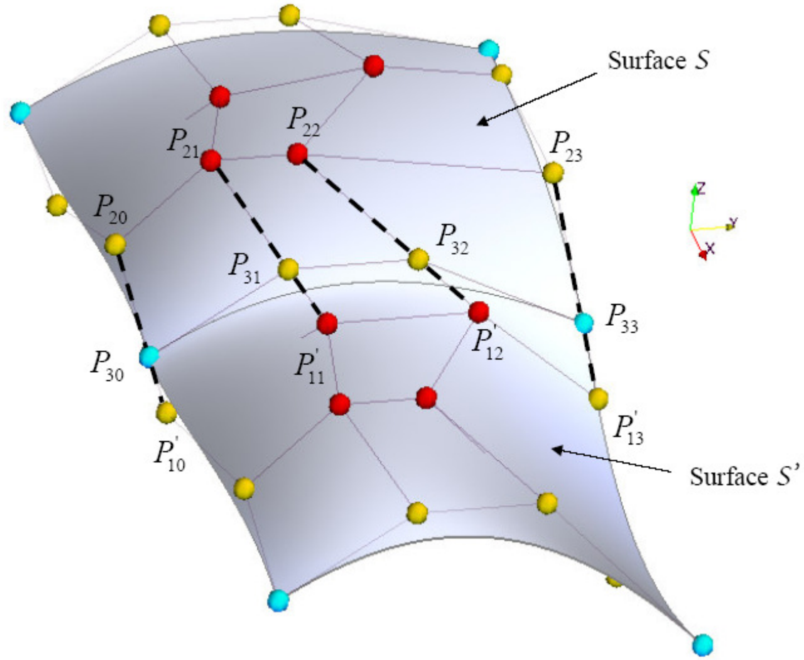
O. Czarny, JCP 2008

- 2D cubic Bezier patch defined by 16 control points

$$\vec{B}(s,t) = \sum_{k,m=0}^{N_1 N_2} \vec{p}_{km} \frac{N!}{k!(N-k)!} s^k (1-s)^{N-k} \frac{N!}{m!(N-m)!} t^m (1-t)^{N-m}$$

- C1 continuity between patches requires that the 4 boundary control points lie on a line with their neighbouring control points





C1 continuity, nodal vectors

- A corner of 4 patches is defined by 9 control points, $p_{ij}=(R,Z,\psi,\ldots)_{ij}$
- Define 3 vectors (equivalence with cubic Hermite elements):

$$\vec{u} = (\vec{p}_{10} - \vec{p}_{00})/h_u, \qquad h_u = \|\vec{p}_{10} - \vec{p}_{00}\|$$

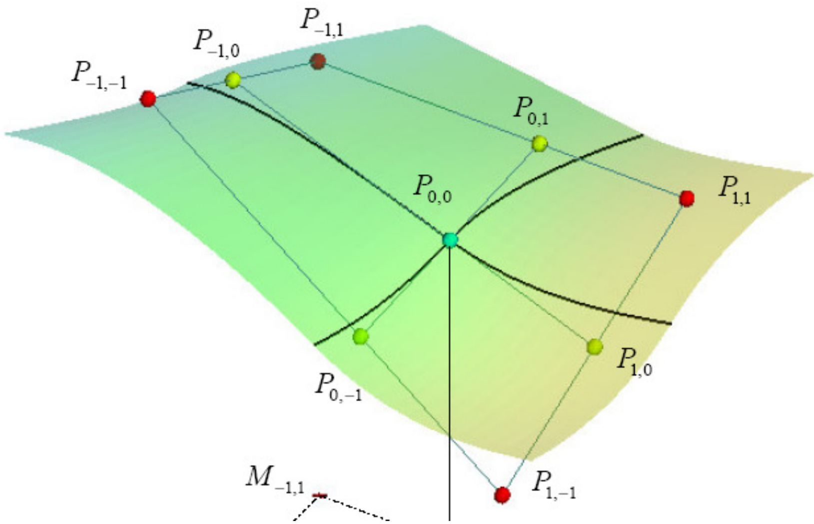
$$\vec{v} = (\vec{p}_{01} - \vec{p}_{00})/h_v, \qquad h_v = \|\vec{p}_{01} - \vec{p}_{00}\|$$

$$\vec{w} = (\vec{p}_{11} + \vec{p}_{00} - \vec{p}_{10} - \vec{p}_{01})/(h_u h_v)$$

$$\frac{\partial \vec{p}_{-1}}{\partial s} = \vec{u}_0 = \tfrac{3}{2}(\vec{p}_{10} - \vec{p}_{00})$$

$$\frac{\partial \vec{p}_{-1}}{\partial t} = \vec{v}_0 = \tfrac{3}{2}(\vec{p}_{01} - \vec{p}_{00})$$

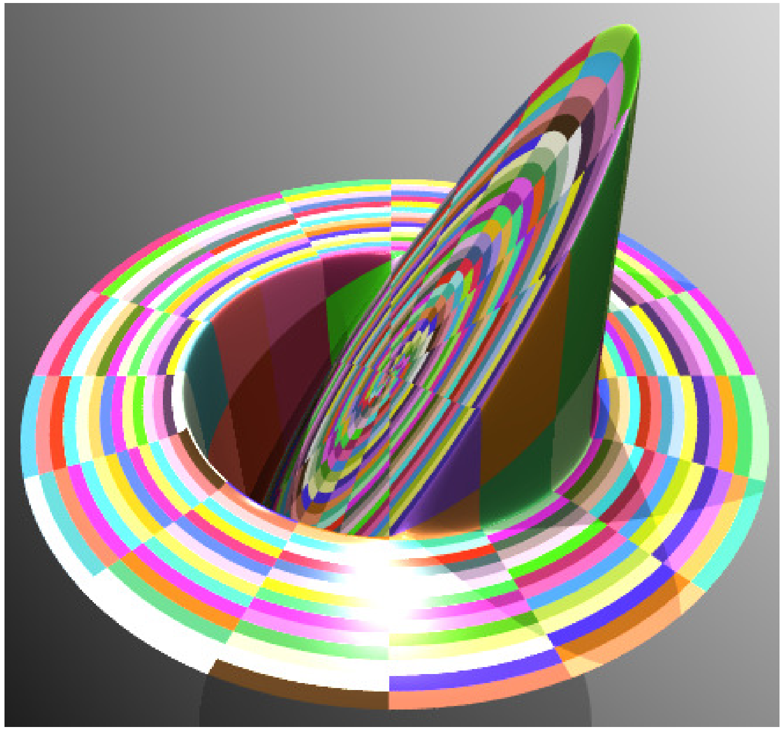
$$\frac{\partial^2 \vec{p}_{-1}}{\partial s \partial t} = \vec{w}_0 = \tfrac{9}{4}(\vec{p}_{11} + \vec{p}_{00} - \vec{p}_{01} - \vec{p}_{10})$$



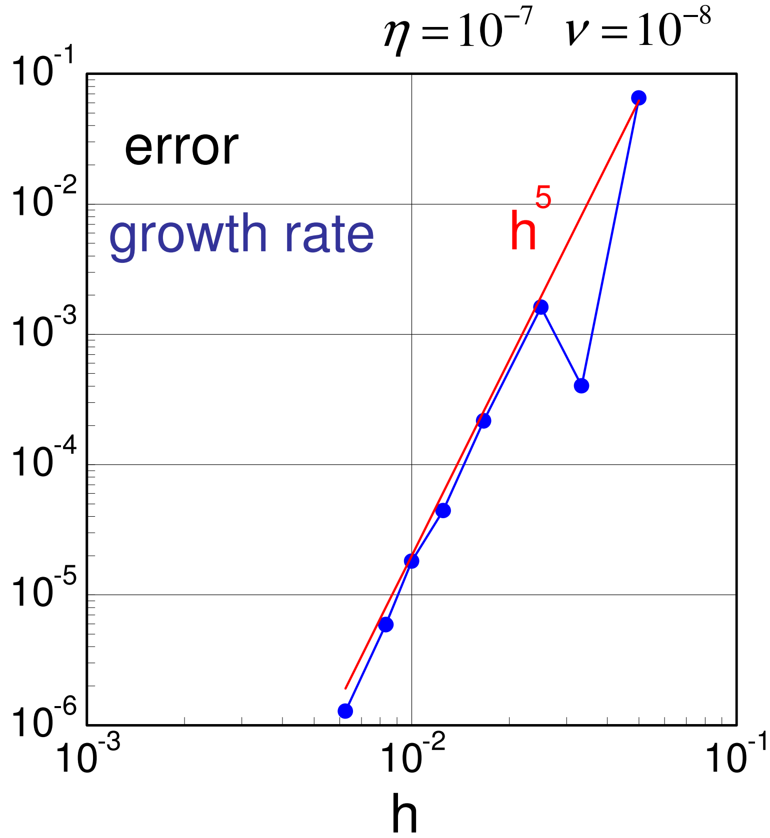
- Continuity in local coordinates (cubic Hermite) requires scale factors h_u and h_v to be the same in the 4 elements
  - too restrictive, i.e. no local refinement

Convergence

- Verification on resistive MHD instabilities:
  - Linear growth rate n=1 resistive internal kink mode :
    - correct scaling error ~ h^5

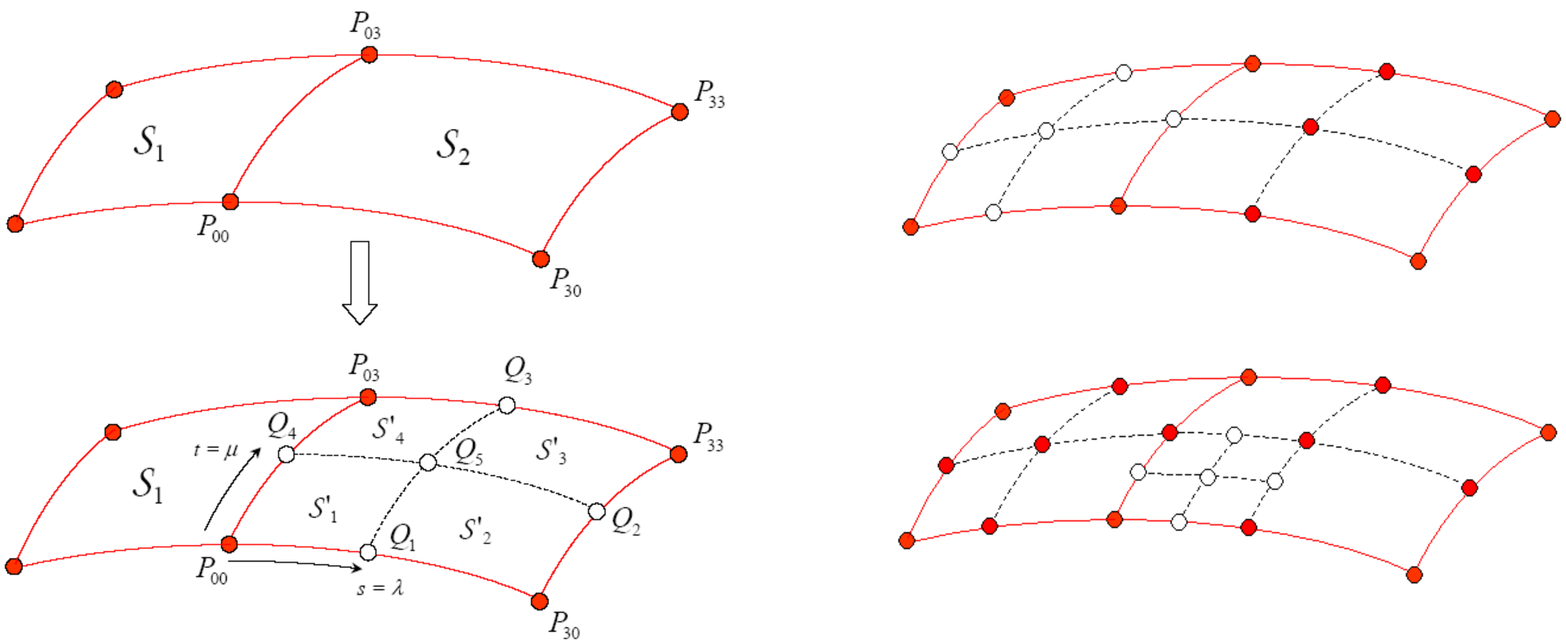


m=1 perturbation internal kink mode



Local Refinement

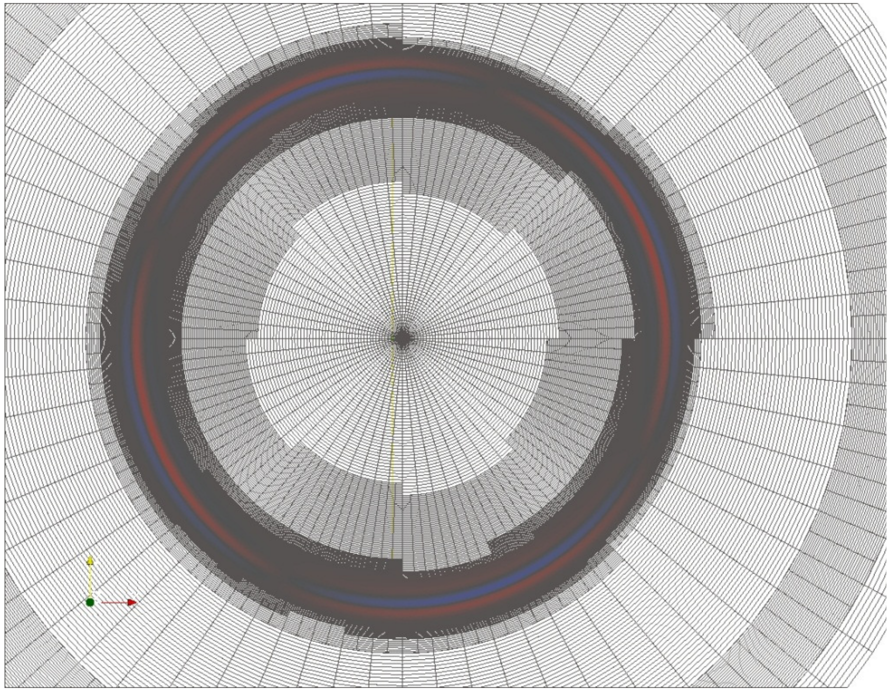
- A Bezier patch can be subdivided into smaller Bezier patches
  - Definition in node vectors and element sizes guarantees C1 continuity
  - Introduces constrained nodes
    - Connectivity matrix



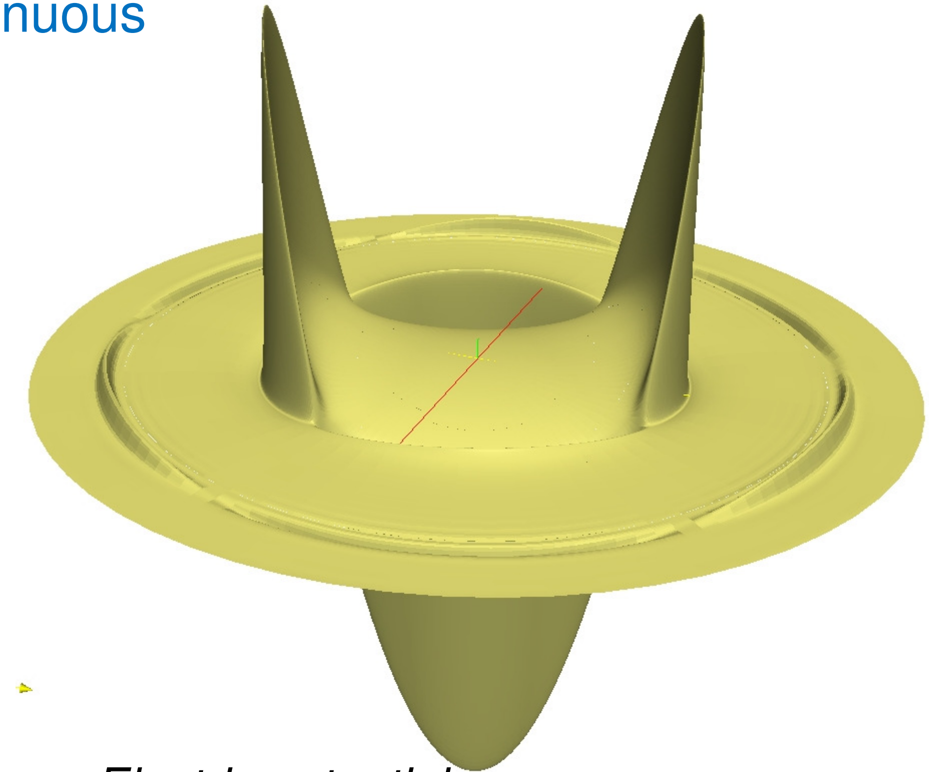
- Choice: a constrained node cannot have a constrained parent
  - Refine neighbouring element to remove the constrained on the parent node

Adaptive Refinement (H. Sellama)

- Refinement of Bezier elements implemented in JOREK
- Tearing mode test case
  - using gradient of current density in refinement criterion
    - formal error based criterion?
  - refined solution remains C1 continuous



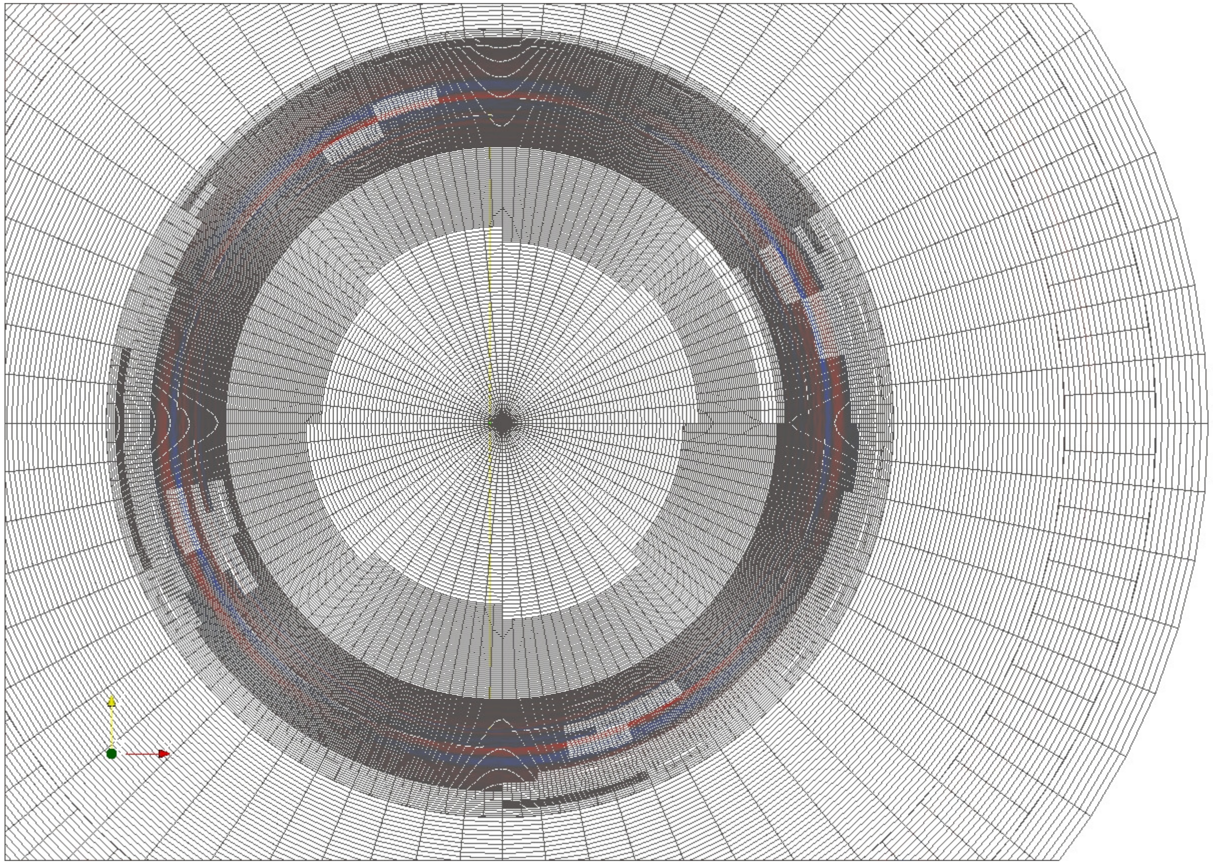
Refined grid (3 levels)



Electric potential

Adaptive Refinement (JOREK)

- Some control of grid regularity is necessary to avoid noise induced by the refinement
  - refine neighbours of an element satisfying the refinement criterion
  - remove single element 'holes'



refined grid (3 levels) without regularity control

JOREK : time stepping

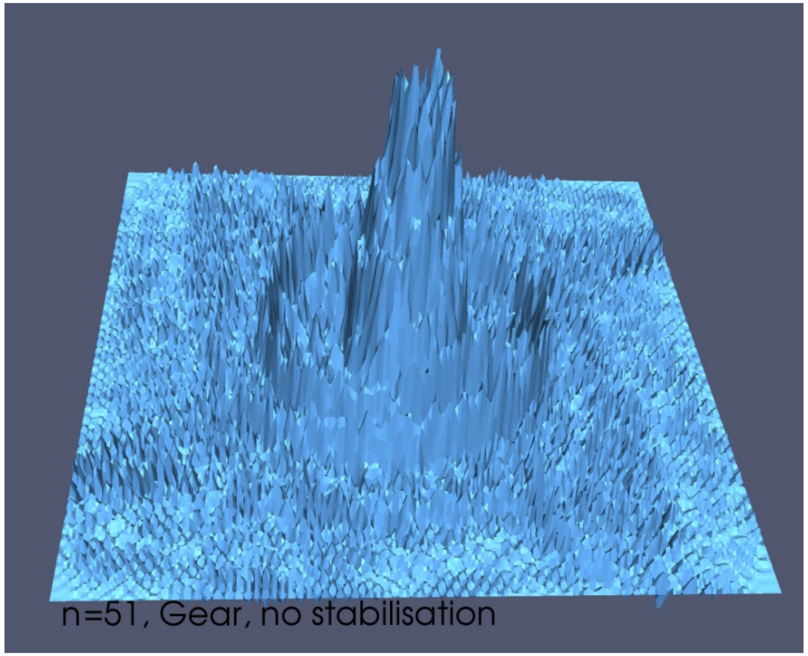
- Fully implicit time evolution allows large time steps:
  - All variables implicitly updated in one step
  - time step independent of grid size
    - 0.5 - 5 Alfven times for ELM simulations to 10.000 Alfven times for slow growing tearing modes
- Linearised Crank-Nicholson scheme (or Gear's scheme):

$$\frac{\partial A(\vec{y})}{\partial t} = B(\vec{y}) \quad \Rightarrow \quad \left(\frac{\partial A(\vec{y}_n)}{\partial y} - \tfrac{1}{2}\delta t \frac{\partial B(\vec{y}_n)}{\partial y} \right) \delta\vec{y} = B(\vec{y}_n)\,\delta t$$

- Leads to very large systems of equations to be solved at every time step
  - sparse matrix solved using iterative method (GMRES)
  - Preconditioning matrices: one for each toroidal harmonic
    - solved using **PaStiX** parallel sparse matrix solver
    - recalculated when GMRES iterations too large
  - Degrees of freedom : up to $2\text{x}10^7$

Stabilisation

- Large (and non resolved) flows may lead to spurious oscillations
  - test case vortex mixing, vorticity equation



$$\frac{\partial w}{\partial t} = -[w, u] + \nu \nabla^2 w$$

- Use finite element stabilisation techniques
  - **Taylor-Galerkin (TG2, TG3)**
  - Galerkin Least Square
  - SUPG (Stream-upwind Petrov-Galerkin)
  - ...

Taylor-Galerkin Stabilisation

- Use higher order time derivatives:

$$\frac{\partial w}{\partial t} = -[w,u] + \nu\nabla^2 w$$

$$w^{n+1} = w^n + \delta t \frac{\partial w^n}{\partial t} + \tfrac{1}{2}\delta t^2 \frac{\partial}{\partial t}\frac{\partial w^n}{\partial t} + \tfrac{1}{6}\delta t^3 \frac{\partial^2}{\partial t^2}\frac{\partial w^n}{\partial t}$$

$$\frac{w^{n+1} - w^n}{\delta t} = -\left[w^n, u^n\right] + \nu\nabla^2 w^n + \tfrac{1}{2}(\delta t)\frac{\partial}{\partial t}\left(-\left[w^n, u^n\right]\right)$$

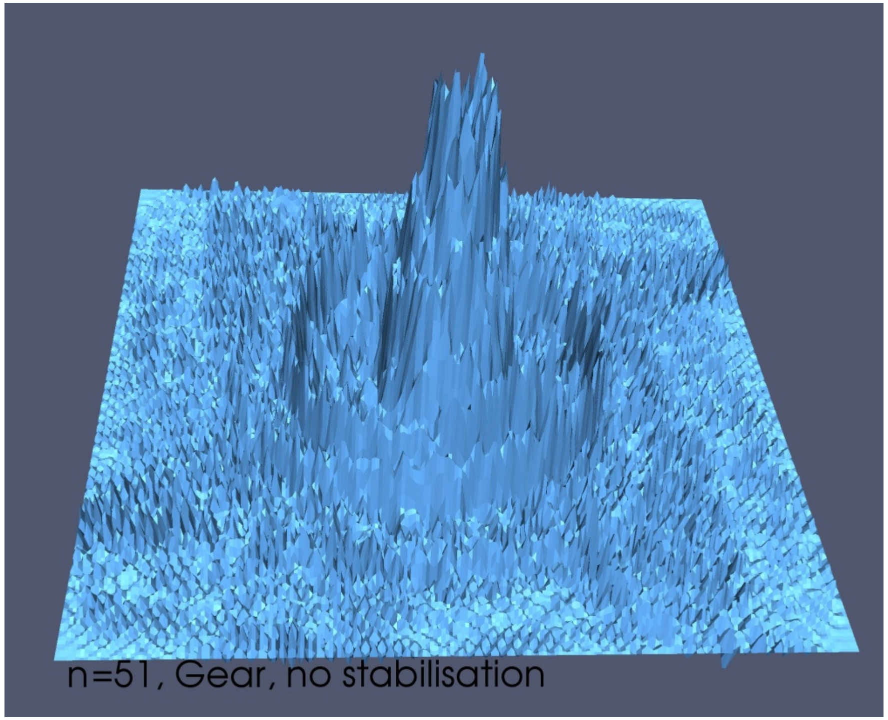
$$\frac{\partial}{\partial t}\left(-\left[w^n, u^n\right]\right) \approx \left[-\left[w^n, u^n\right], u^n\right]$$

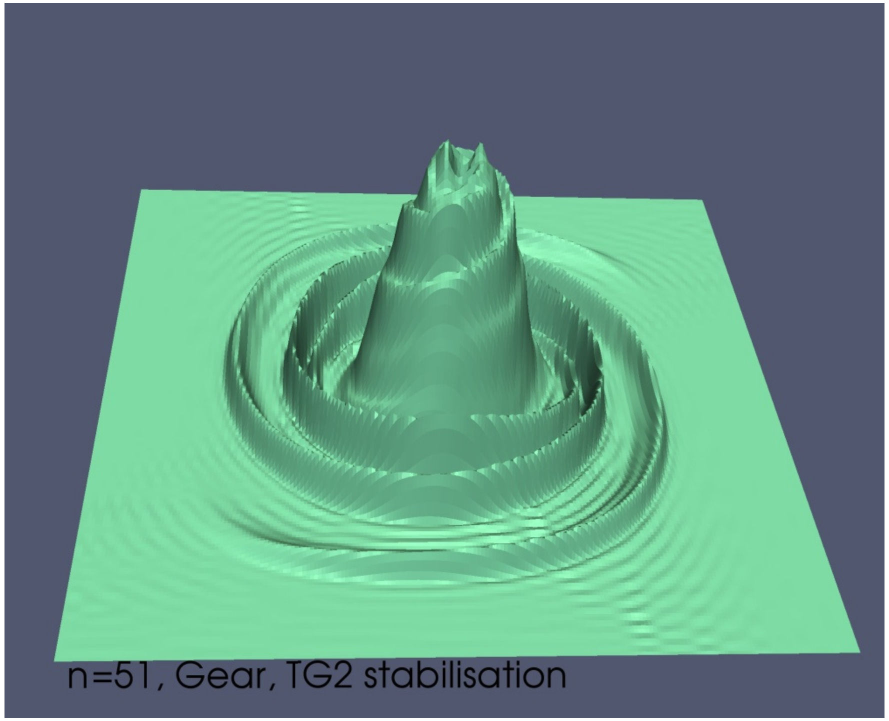
- Weak form (implicit TG2):

$$v^* \frac{\delta w}{\delta t} = -v^*\left(\left[w^n, u^n\right] + \nu\nabla^2 w^n - \tfrac{1}{2}\left[\delta w, u^n\right] - \tfrac{1}{2}\left[w^n, \delta u\right] + \tfrac{1}{2}\nu\nabla^2 \delta w\right)$$
$$+ \tfrac{1}{4}\delta t\left[w^n, u^n\right]\left[v^*, u^n\right] + \tfrac{1}{8}\delta t\left[w^n, u^n\right]\left[v^*, \delta u\right] + \tfrac{1}{8}\delta t\left[\delta w, u^n\right]\left[v^*, u^n\right] + \tfrac{1}{8}\delta t\left[w^n, \delta u\right]\left[v^*, u^n\right]$$

TG2 Stabilisation

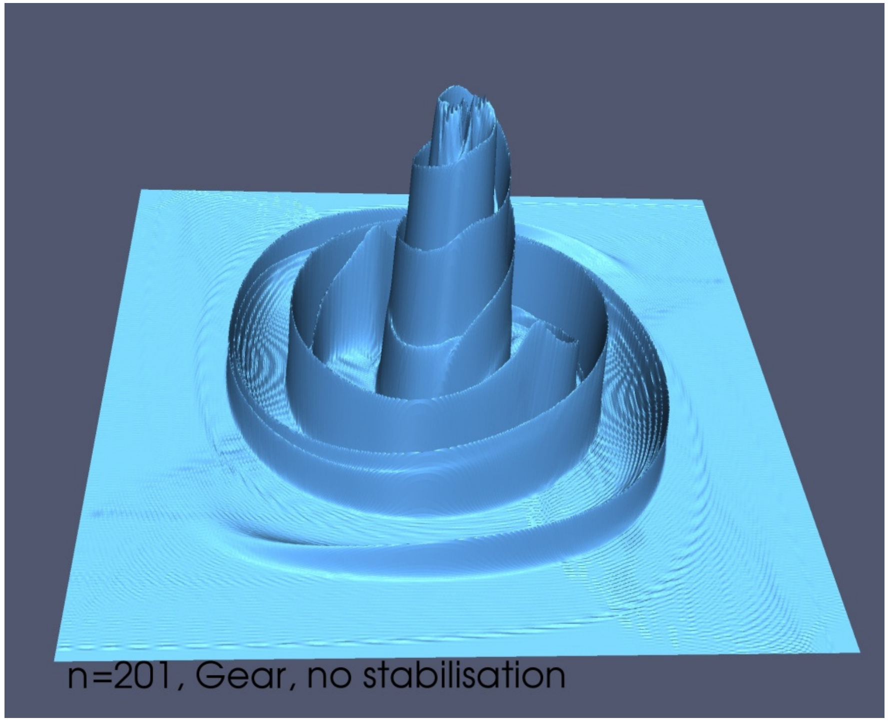
- Stabilisation gives large improvement for this test case

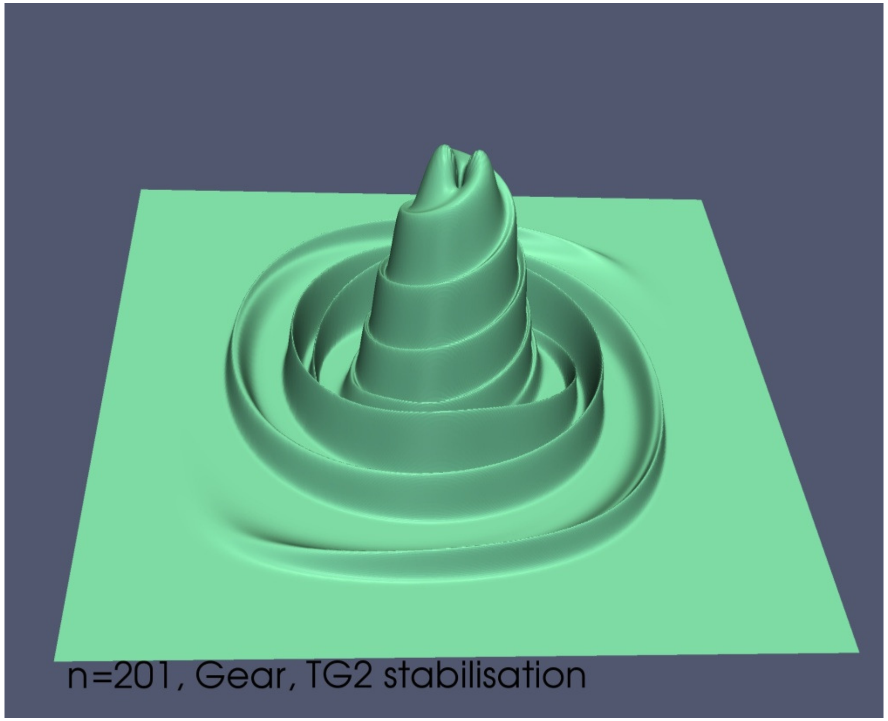




TG2 Stabilisation

- At high resolution TG2 stabilisation may be too strong





- TG2 stabilisation implemented (and used) in JOREK
- Also hyper-diffusion terms

JOREK Parallelisation

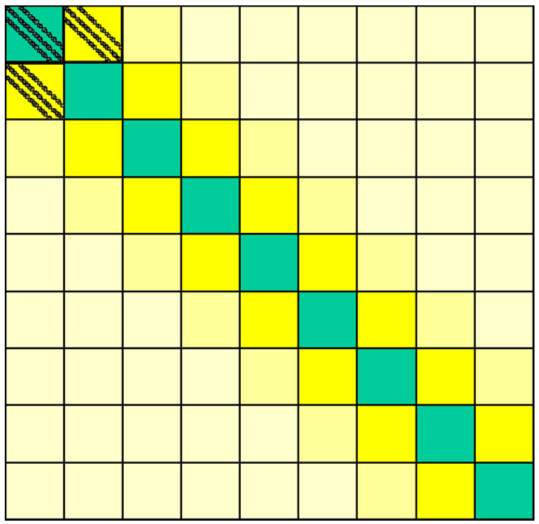
- JOREK uses MPI and OPENMP
  - Parallelisation is necessary for both CPU and memory requirements
    - up to 2000 cores
  - Matrix construction:
    - Distribution (MPI) of finite elements over nodes
    - Using threads (OPENMP) inside each node:

```
!$omp do
do ife = 1, n_local_elms
   call element_matrix(ELM,…)
   !$omp critical
   call add_element_to_matrix(ELM)
   !$omp end critical
enddo
!$omp enddo
```

    - very good scaling
    - option: using MURGE library: http://murge.gforge.inria.fr/files/include/murge-h.html

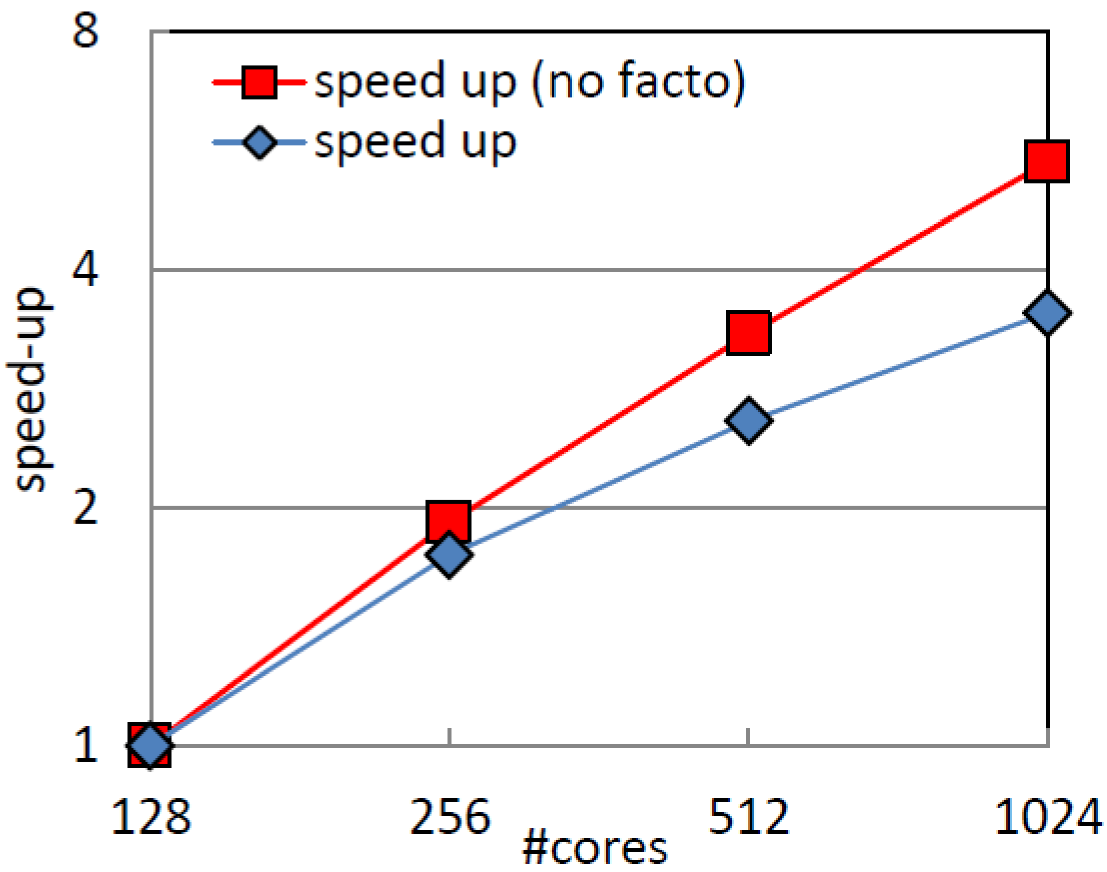
JOREK Parallelisation

- Matrix solution:
  - Preconditioned GMRES:
    - Use sub-matrices of each toroidal harmonic as preconditioner
      - divides factorisation of preconditioning matrix into N independent parts
        - » Block-Jacobi preconditioning
      - each Factorisation and Solve parallelised using N instances of PaSTiX sparse matrix library
        - » http://pastix.gforge.inria.fr/files/README-txt.html
      - Factorisation only done when number of GMRES iterations > 20-50
    - GMRES:
      - Matrix vector multiplication (MPI/OPENMP)
      - Matrix solve (PaStiX)
    - Parallelisation scaling is challenging
      - common for implicit fluid codes



Parallelisation Scaling

- JOREK strong scaling:
  - clock time as a function of cores at fixed problem size



JOREK Activities

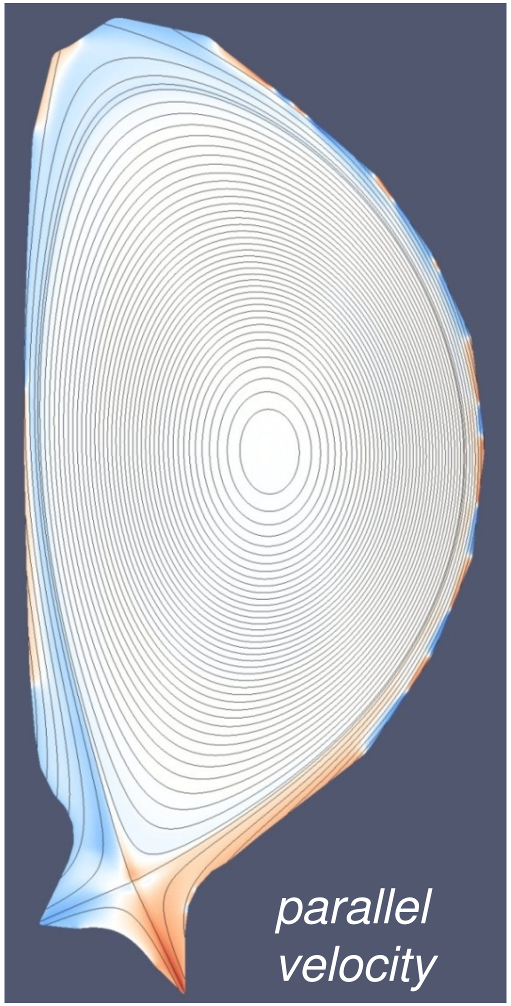
- Disruption simulation, massive gas injection
  - Alexandre Fil (CEA), Eric Nardon (CEA)
- ELM simulations, ELM control RMPs
  - Francois Orain (CEA), Marina Becoulet (CEA), Jorge Morales (CEA), Stanislas Pamela (UKAEA), Matthias Holzl (IPP), Guido Huijsmans (ITER)
- ELM control, pellets
  - Shimpei Futatani (Barcelona)
- ELM control, QH-mode
  - Feng Liu (ITER)
- Tearing mode control, current drive
  - Egbert Westerhof (FOM), Jane Pratt (UK)
- VDEs
  - Matthias Holzl (IPP), Eric Nardon (CEA), Ksenia Aleynikova (Moskou)
- Numerical schemes
  - Boniface N'Konga (Nice), Emmanuel Franck (IPP), Ahmed Ratnani (IPP)
- Extended MHD models
  - ...

ELM simulations in ITER geometry

- Evaluation of parallel fluxes to divertor and first wall requires open field lines crossing divertor *and first wall*
  - Finite element grid extended to first wall panels
  - Bohm boundary conditions on all surfaces:

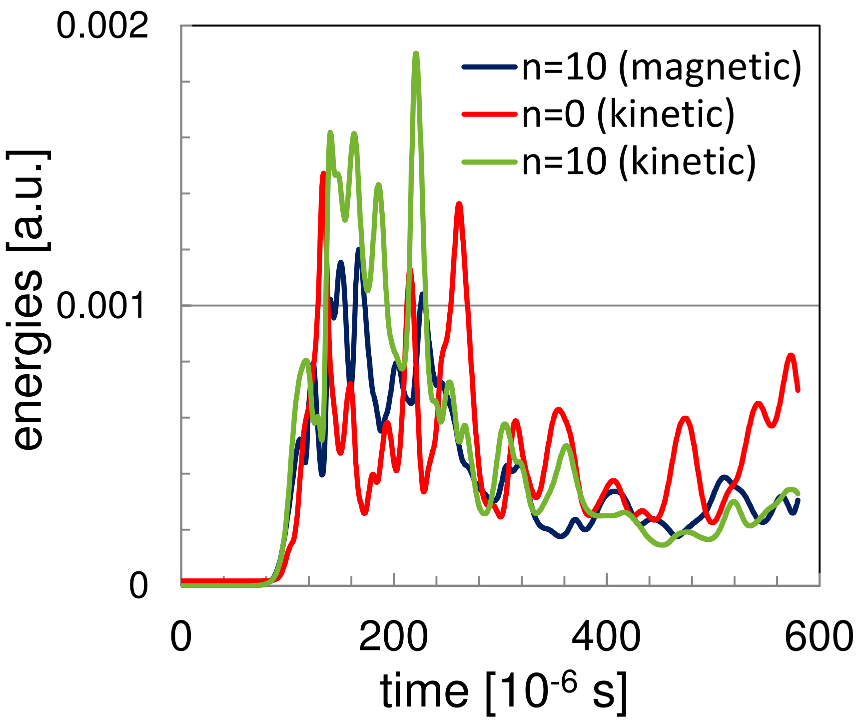
$$v_{\parallel} = c_s \qquad K_{\parallel}\vec{b} \cdot \nabla T = (\gamma - 1) n T c_s$$

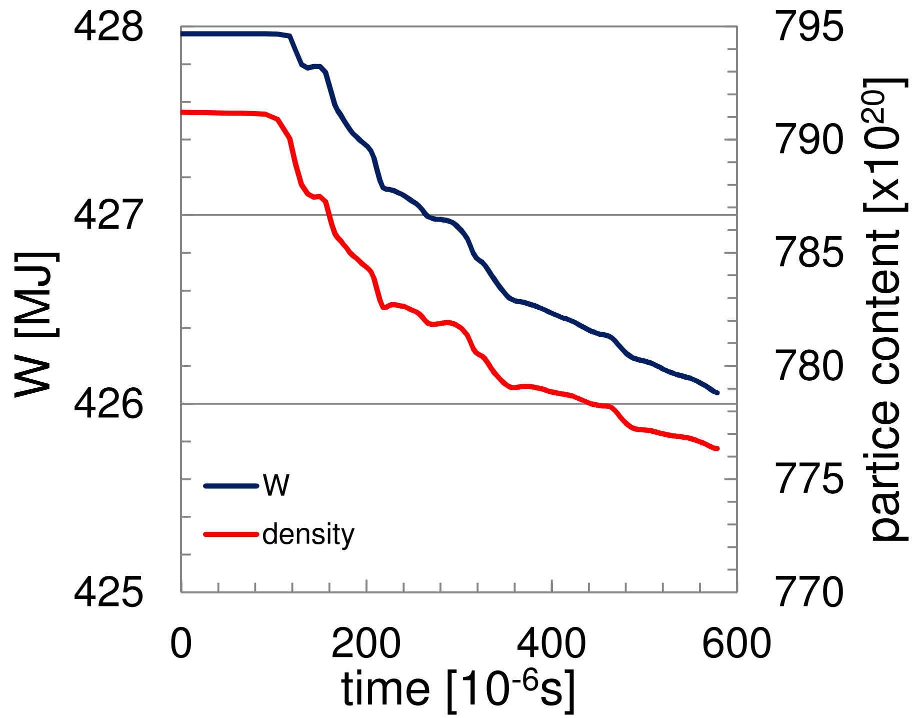
  - change of sign $V_{//}$ at points where magnetic field is parallel to wall (outflow only)
  - leads to local density maxima on the wall at $V_{//}=0$
- Stationary equilibrium on millisecond time scale, not transport time scale
  - (quasi-) Stationary parallel and poloidal flows



ELM Simulation ITER Q=10 Scenario

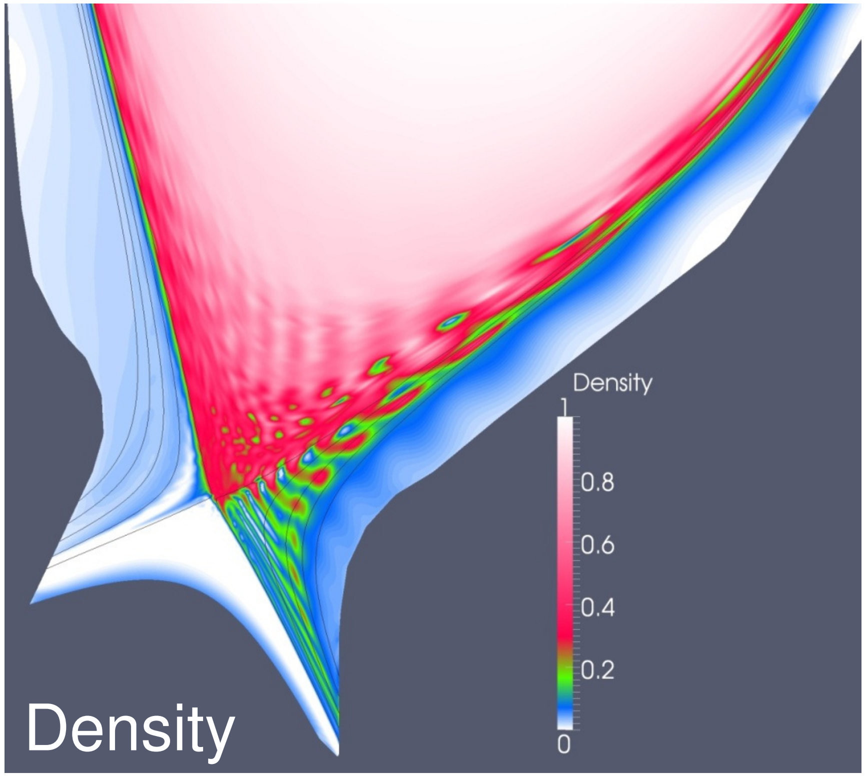
- Equilibrium based on Corsica scenario “5.5keV”
  – T_{ped}=5.2 keV, N_{ped}= $6x10^{19}$ m^{-3} , δ_{ped}=6cm, I=15MA , (S=10^6)
- Energy evolution (n=0,10):
  – Small convective ELMs, duration ~200 μs
  – comparable amplitude magnetic and kinetic perturbations
  – ΔW = 2MJ, ΔW/W = 0.5%, Δn/n = 2 %

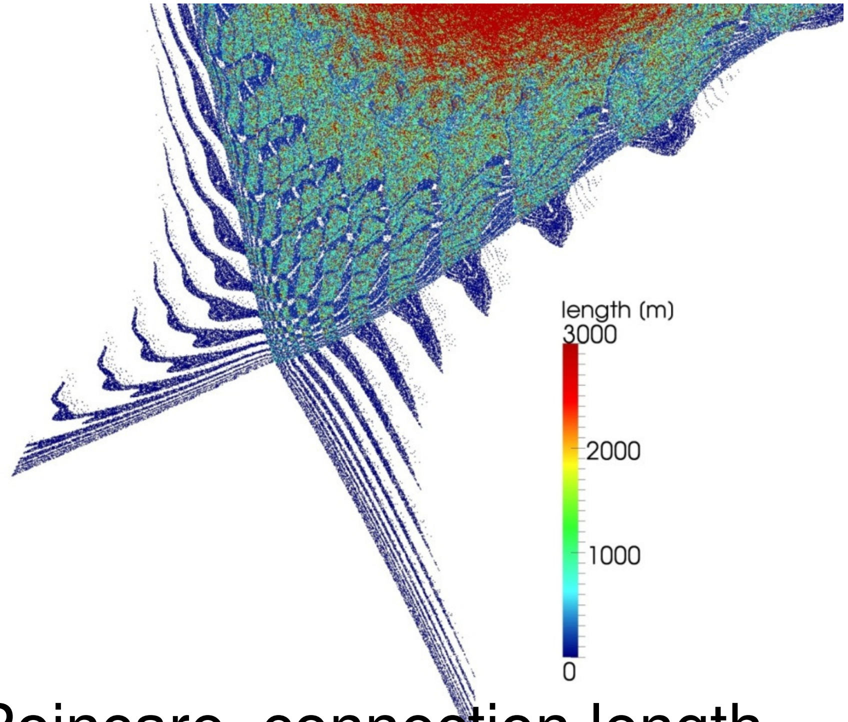




ELM Energy Losses

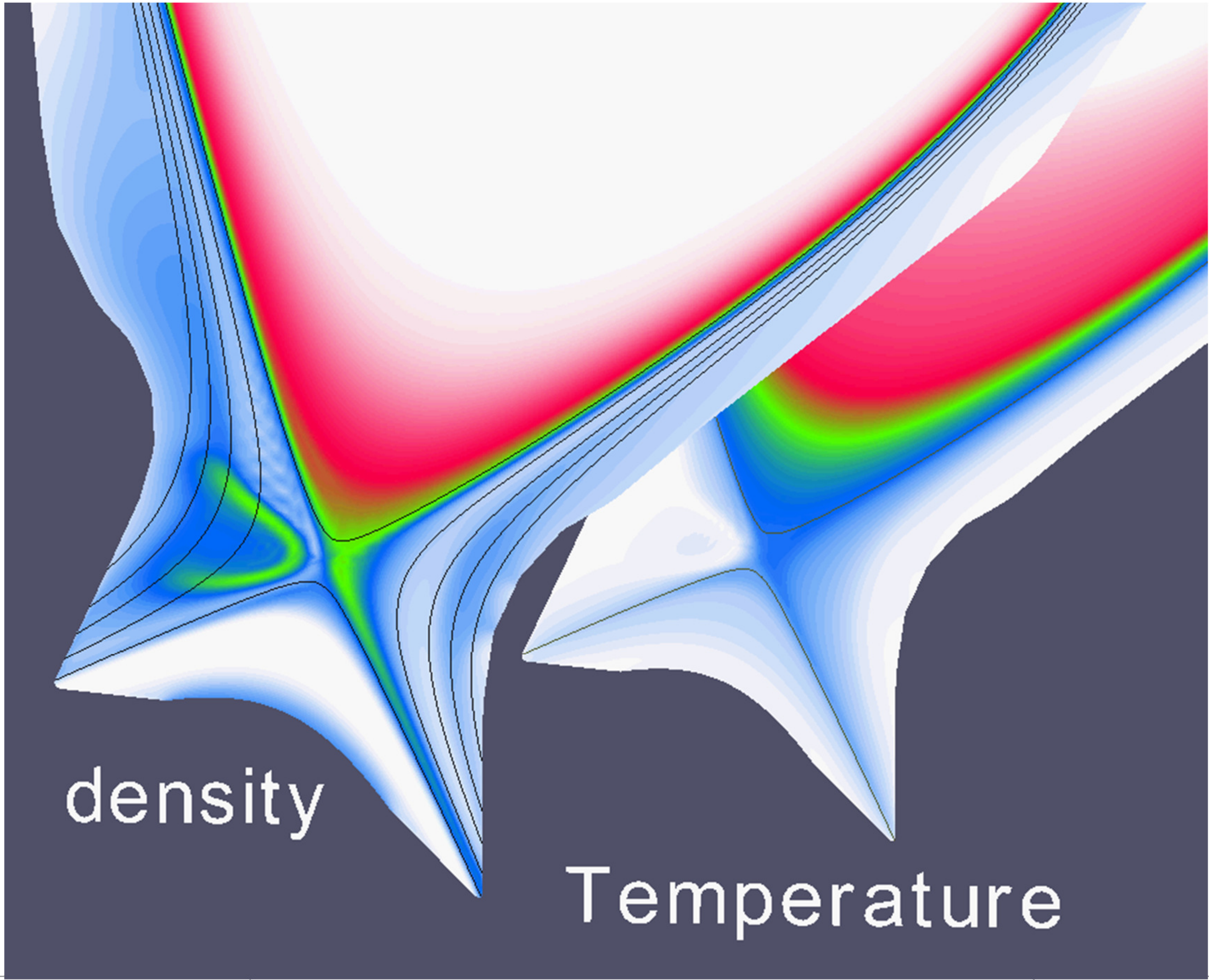
- Energy loss during ELMs is due to two mechanisms:
  - Formation of filaments : convective energy loss
  - Formation of magnetic tangles : conduction parallel to field lines





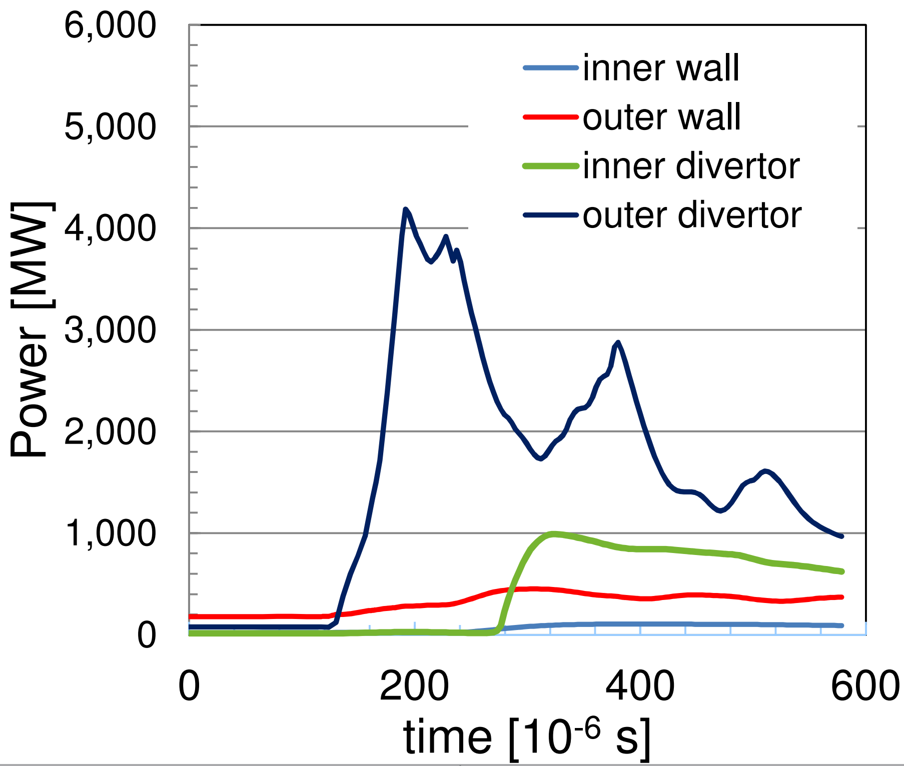
Poincare, connection length

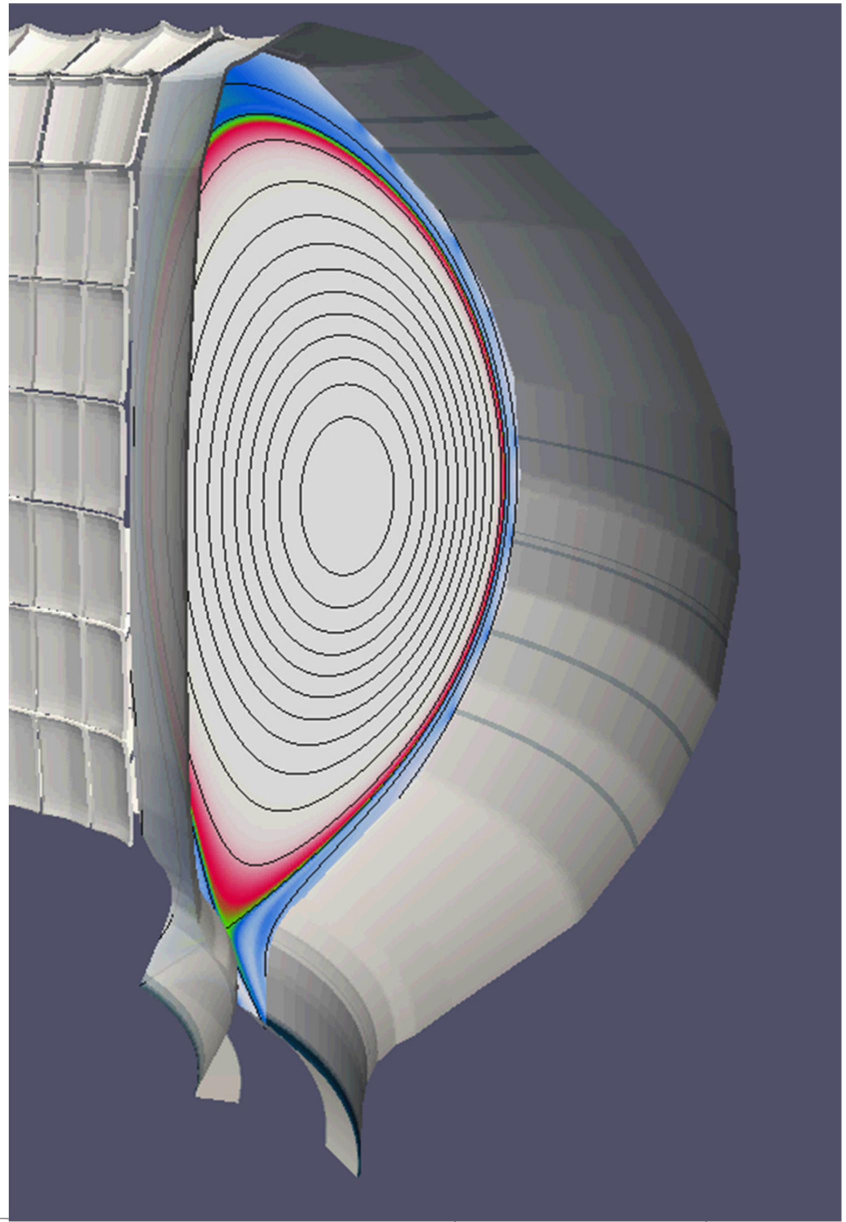
Filaments



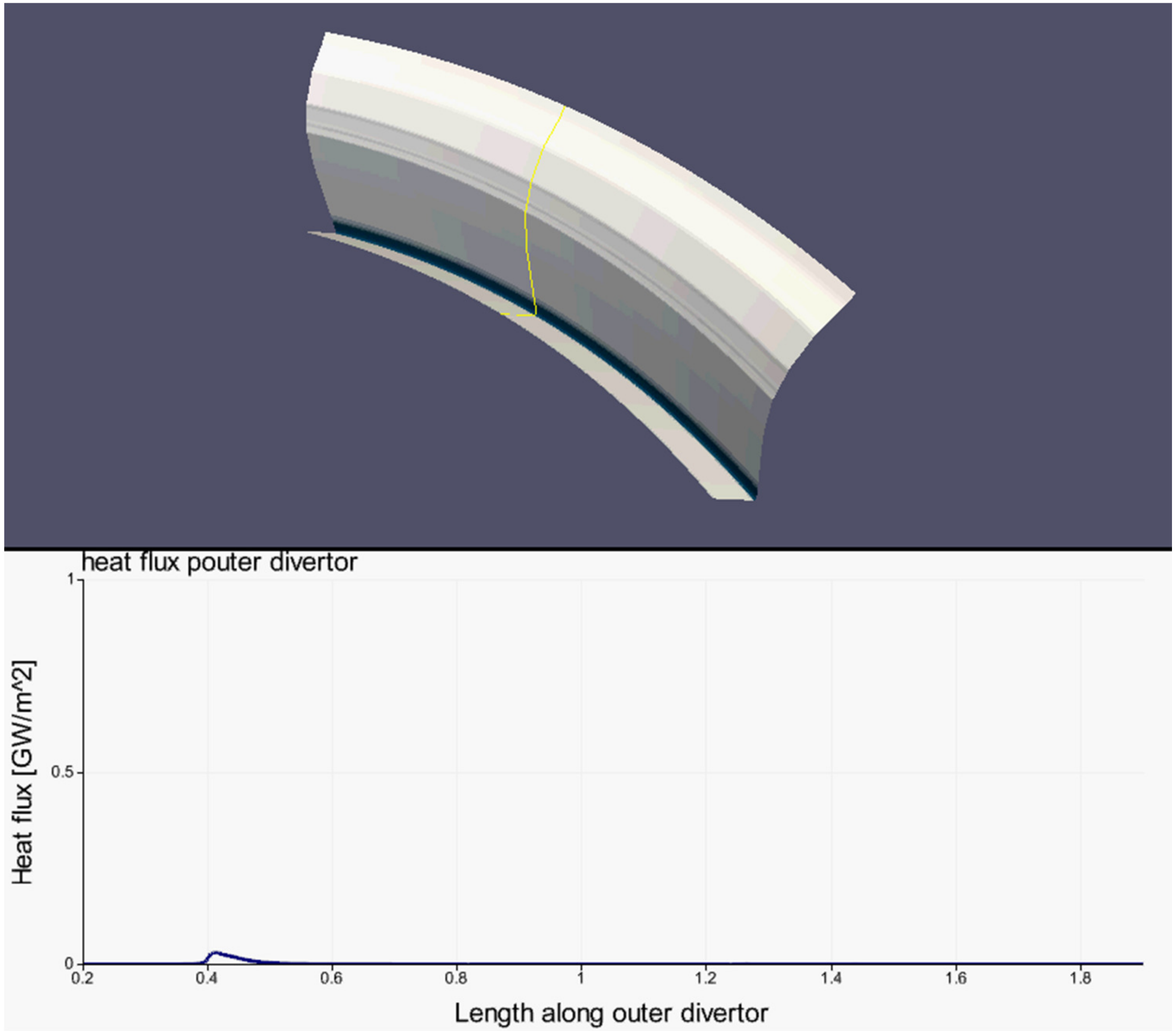
ELM Energy Loss Distribution

- Parallel energy flux into the wall/divertor
  - mostly due to parallel losses along filaments on unperturbed field lines
  - delay between outer and inner divertor
    - consistent with parallel convection time
  - power to first wall <10% of divertor





Outer Divertor Heat Flux



Plasma-Wall-Vacuum

- Plasma
  - Reduced or full MHD

$$E = -\nabla\varphi - \partial A/\partial t$$

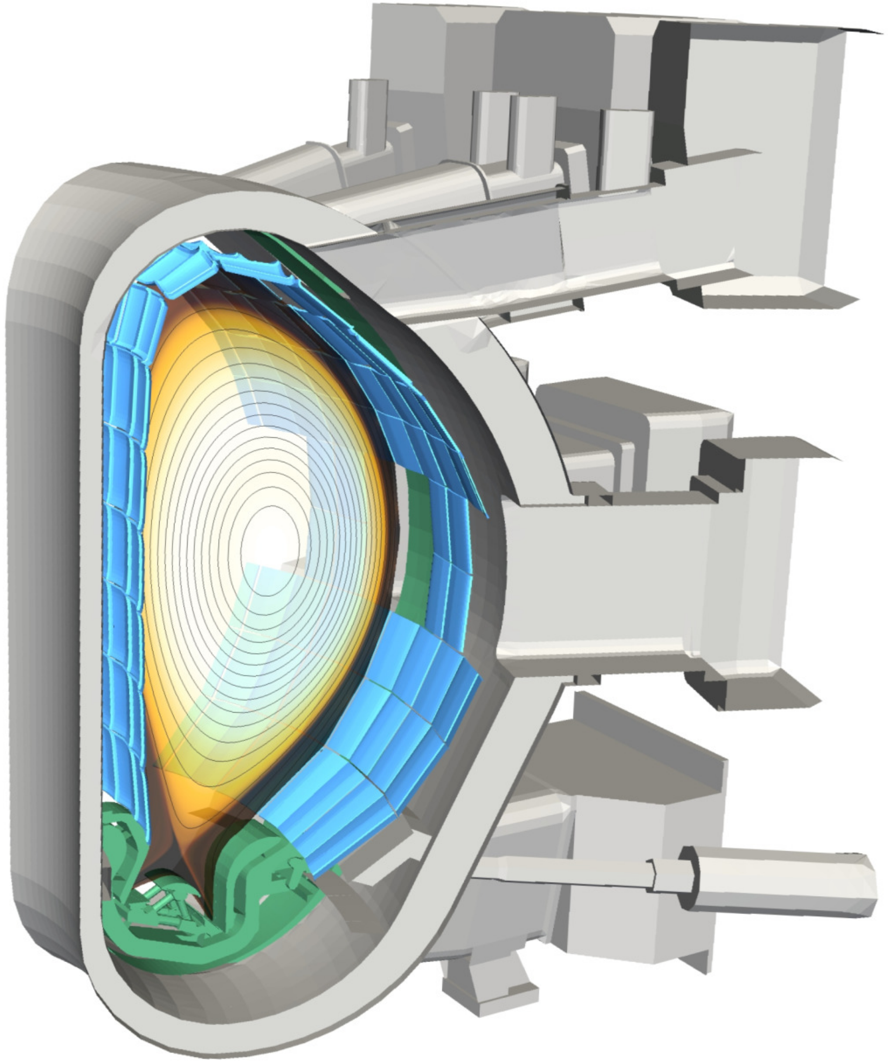
- Conducting structures (coils)
  - Some in contact with plasma

$$J = \sigma E \qquad E = -\nabla\varphi - \partial A/\partial t$$

$$\nabla \times \nabla \times A = -\sigma(\nabla\varphi + \partial A/\partial t)$$

- Vacuum

$$\nabla \cdot B = 0$$
$$\nabla \times B = \nabla \times (\nabla \times A) = 0$$



Plasma-Wall-Vacuum

- JOREK-STARWALL
  - Plasma: Ohms Law: $$\frac{\partial\psi}{\partial t}=\eta R^2\nabla\cdot\left(\frac{1}{R^2}\right)\nabla_\perp\psi-\vec{B}\cdot\nabla\Phi$$
    - Weak form:

$$\int\frac{1}{R^2}\psi^*\frac{\partial\psi}{\partial t}dV=\int\psi^*\eta\nabla\cdot\left(\frac{1}{R^2}\nabla_\perp\psi\right)-\frac{1}{R^2}\psi^*\vec{B}\cdot\nabla\Phi dV$$

$$=\int\frac{1}{R^2}\nabla\left(\eta\psi^*\right)\cdot\nabla_\perp\psi dV-\oint\psi^*\eta\frac{1}{R^2}\left(\nabla_\perp\psi\cdot\vec{n}\right)dS-\int\frac{1}{R^2}\psi^*\vec{B}\cdot\nabla\Phi dV$$

  - Vacuum solution (STARWALL) yields relation tangential to normal magnetic field at the computational boundary:

$$\nabla\psi\cdot\vec{n}=M\left(\nabla\psi\times\vec{n}\cdot\vec{e}_\varphi\right)$$

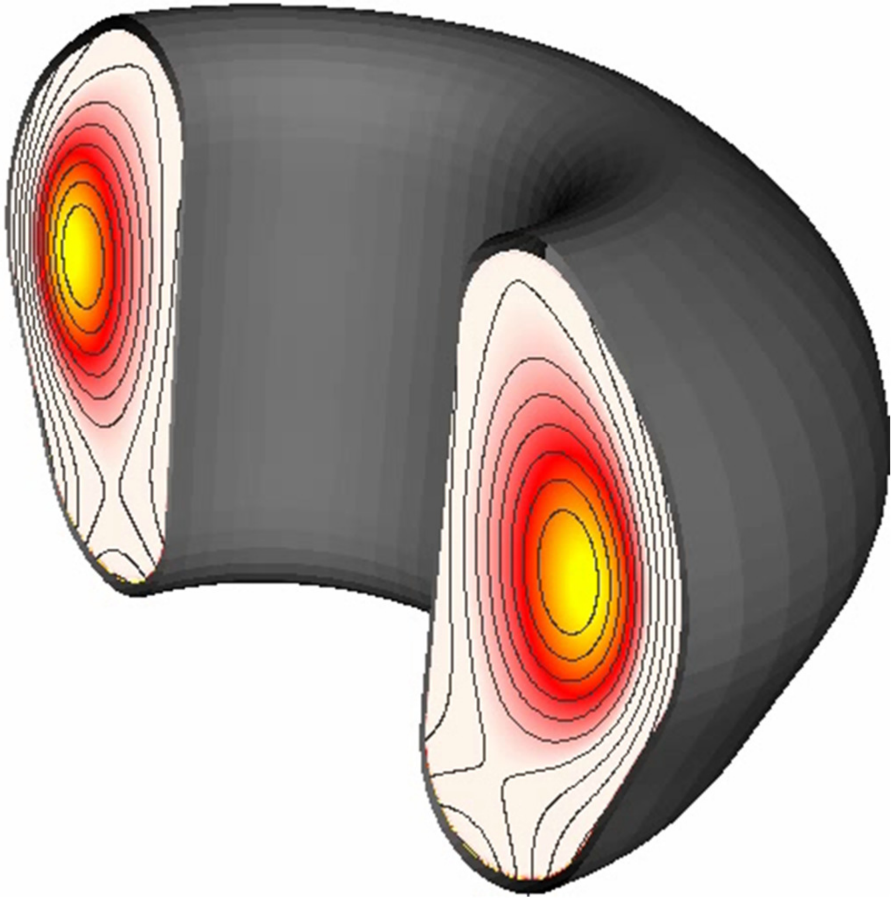
  - Insert in weak form:
    - Natural (Neumann) boundary condition will automatically be satisfied

$$\int\frac{1}{R^2}\psi^*\frac{\partial\psi}{\partial t}dV=\int\frac{1}{R^2}\nabla\left(\eta\psi^*\right)\cdot\nabla_\perp\psi dV-\oint\psi^*\eta\frac{1}{R^2}M\left(\nabla\psi\times\vec{n}\cdot\vec{e}_\varphi\right)dS-\int\frac{1}{R^2}\psi^*\vec{B}\cdot\nabla\Phi dV$$

  - Eddy currents only, new scheme for halo currents is required

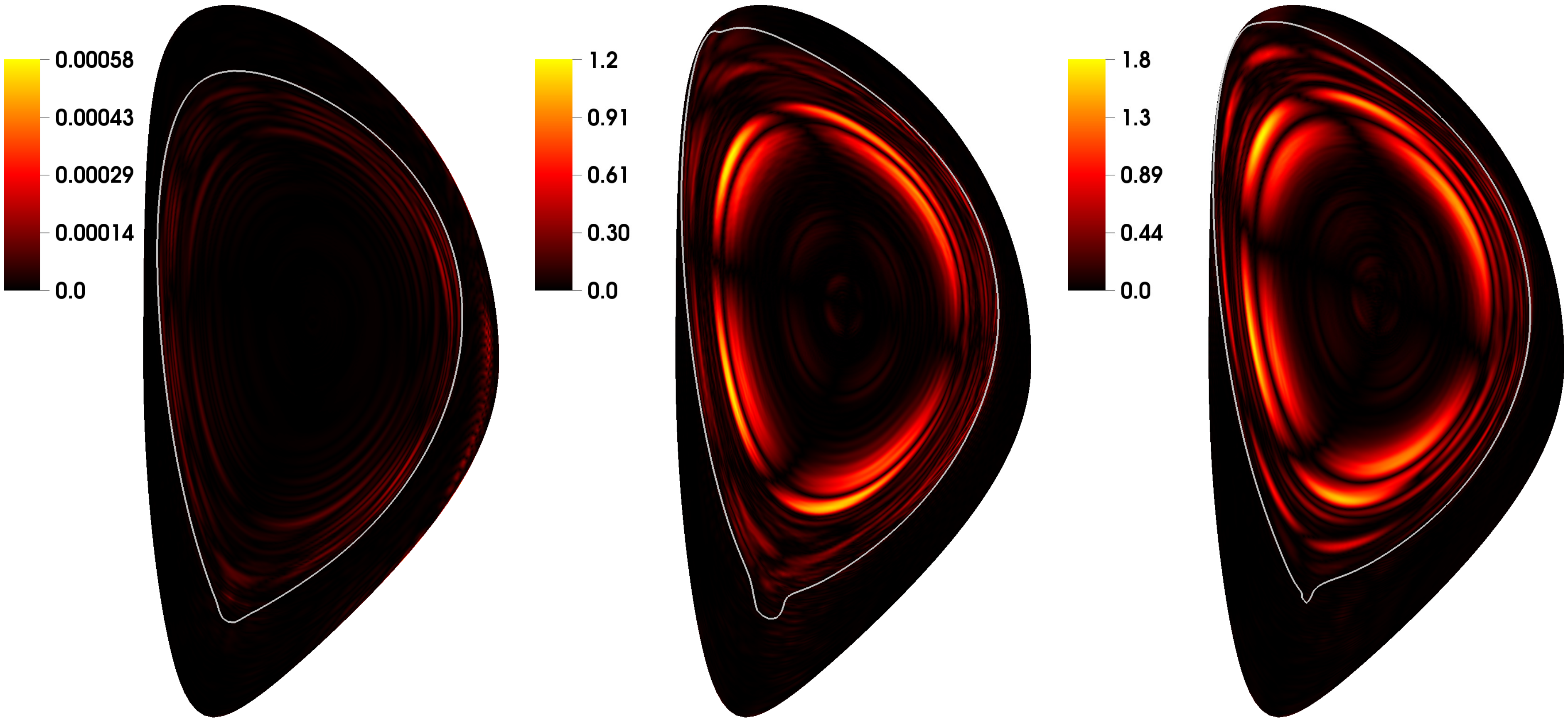
Simulation of VDEs *(M. Holzl, IPP)*

- VDE simulation in ITER (JOREK)



3D VDE *(K. Aleynikova, MIPT)*

- First 3D VDE Simulation in ITER geometry:
  - time scale ~5 ms

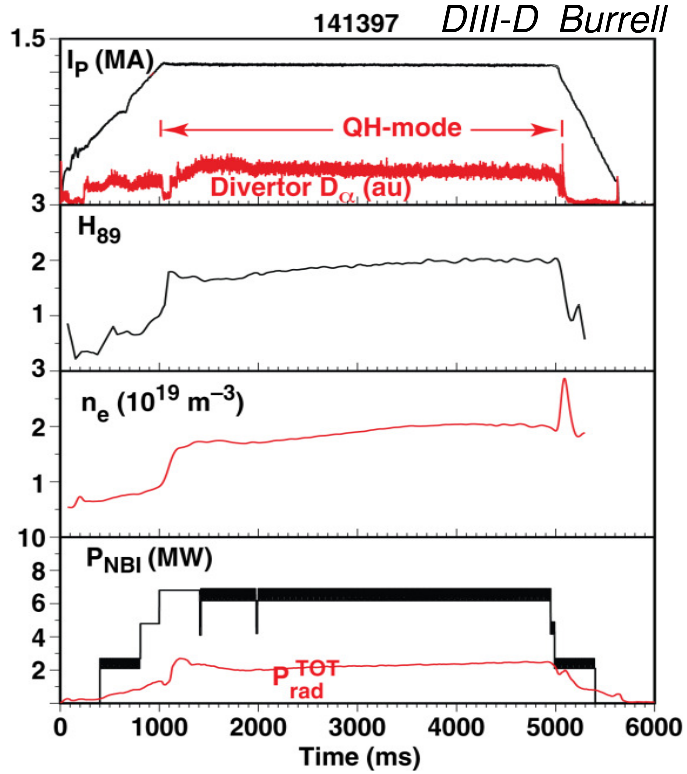


n=1 current perturbation

QH mode Plasmas

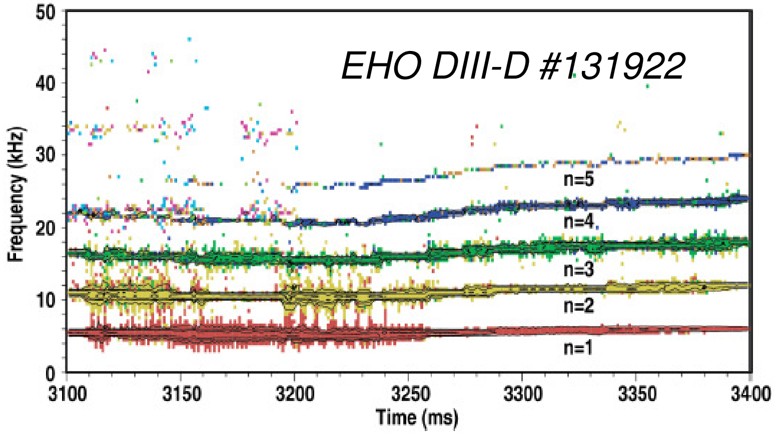
- ITER H-mode scenario is expected have have large Edge Localised Modes (ELMs) leading to large transient heat loads
  - ELMs will be controlled using magnetic perturbations (RMPs) or D_2 pellet injection

- Possible alternative : QH-mode plasma
  – H-mode confinement
  – ELM-free (no transient divertor heat loads)
  – Edge Harmonic Oscillation (EHO) causes density loss and steady state H-mode



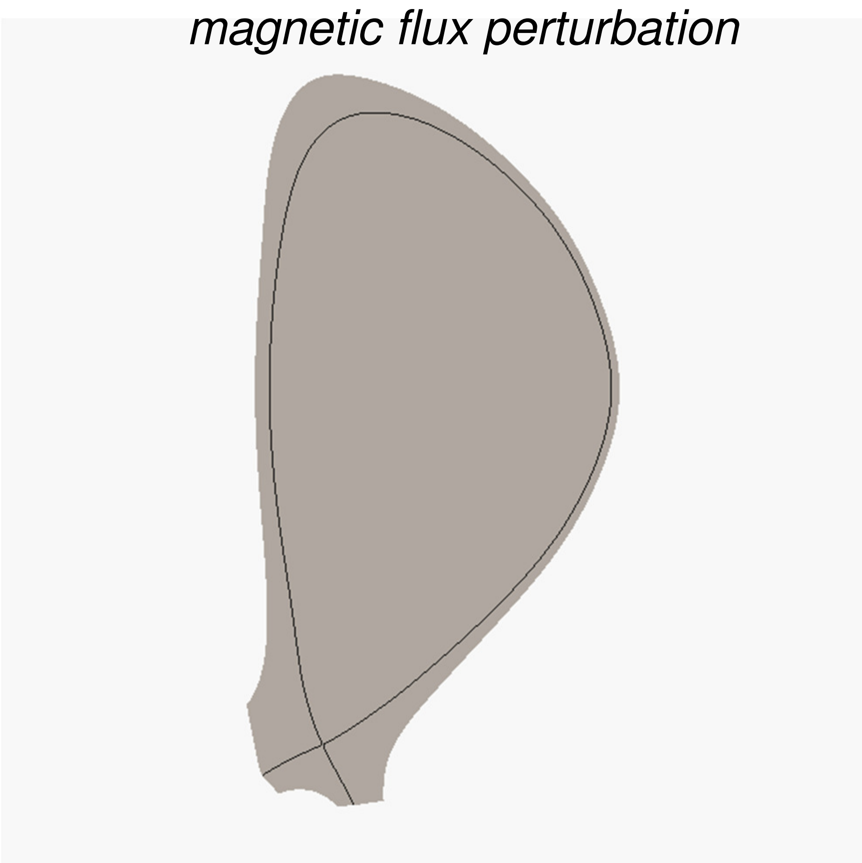
Is QH-mode be a viable option for ITER?

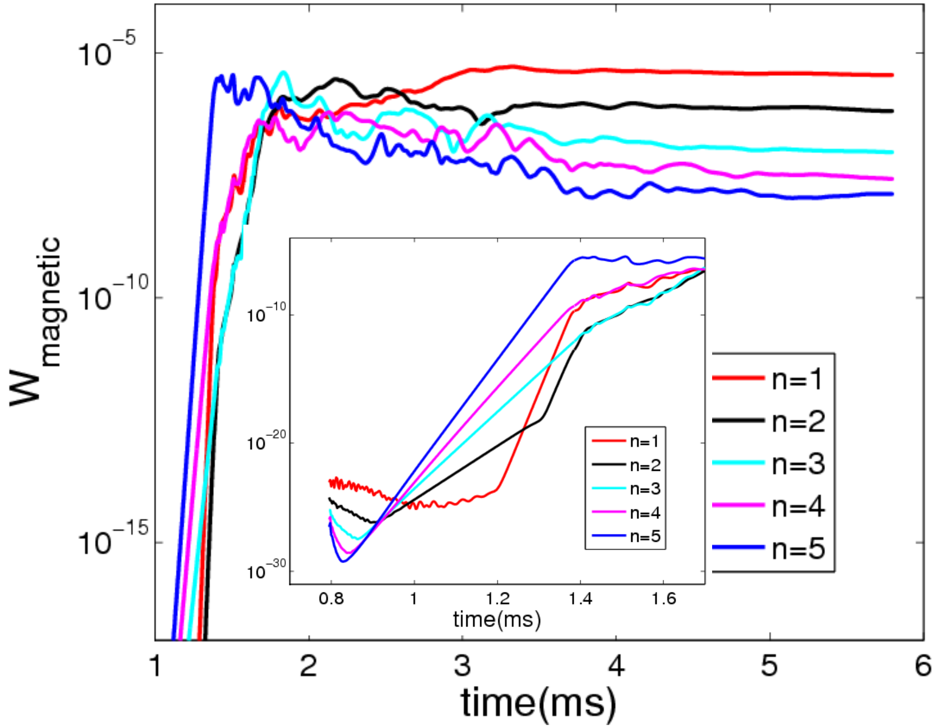
- Validation of Non-linear MHD simulations on DIII-D tokamak
- Extrapolation to ITER



MHD Simulations of QH-mode *(F. Liu, ITER)*

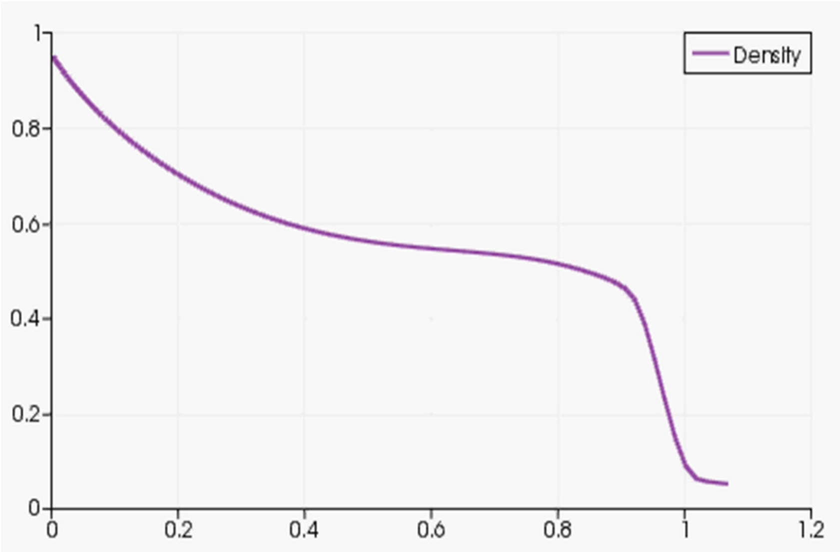
- Simulation starting from initial state from DIII-D QH-mode plasma shows a growing external kink instability
  - External kink mode saturates non-linearly into a new quasi-stationary 3D state

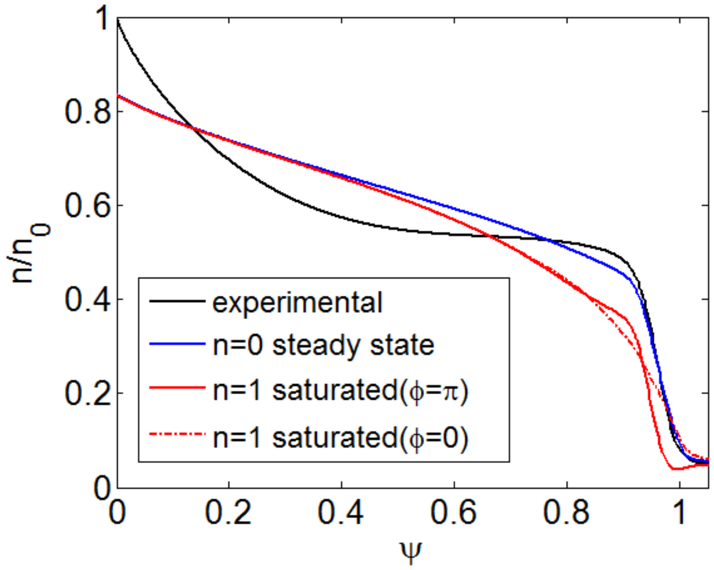


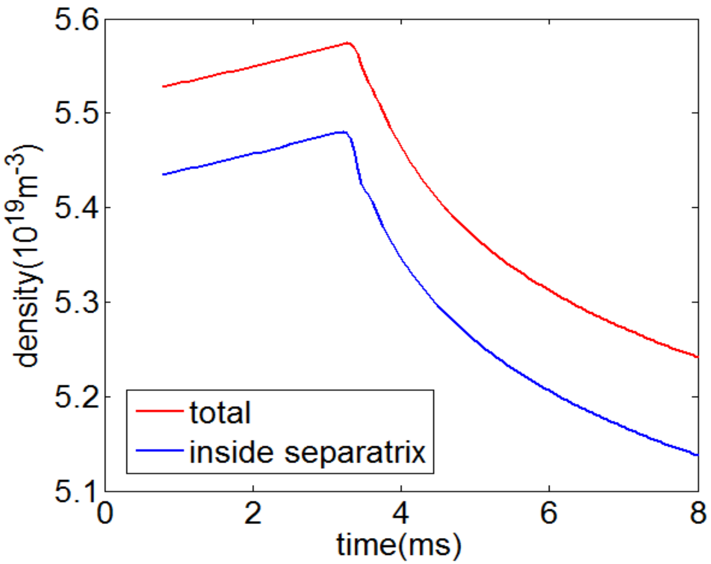


Density Losses due to External Kink Mode

Density profile time evolution:







- Saturated Kink mode leads to increased density losses:
  - Pedestal density reduced by 25%
  - Total density by ~10%
- Temperature not affected
- Qualitative agreement with experiment

Future directions

- Aim: ITER simulation in realistic geometry/plasma parameters
  - Disruption (VDE) simulations and control methods
  - ELM and ELM control in ITER plasmas
- Extended MHD models
  - Reduced or full MHD (gyrofluid)
  - Comparison of models
- Including radiating impurities
  - Fluid or discrete particles
- Including halo currents
- Interaction with particles
  - Runaway electrons
  - Fast ions (fusion alphas, heating)
  - impurities
- Numerics
  - 3D FEM, splines
  - Solvers, scalability

Conclusions

- High priority ITER issues are related to MHD instabilities (Disruptions, ELMs and their control)
  - Extrapolation from current experiments to ITER requires validation of MHD simulations (i.e. comparison of simulations with experimental observations)
  - MHD simulations also important for physics understanding

- MHD simulations need to be more and more realistic
  - Extended MHD models
  - Exact geometry (use CAD models) interaction with conducting structures
  - Description of detached divertor